Enhancing an Early Inclusive Mathematical Learning? Investigating Cubarithm's Semiotic Potential for Children with Visual Impairments

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Introduction

Mathematics stands as a foundational pillar of education, fostering critical thinking, problem-solving, and cognitive development (OECD, 2023). However, for children with visual impairments, the path to mathematical learning—or often just its accessibility—presents unique and daunting challenges. The reliance on visual representations in traditional mathematical education has created barriers that can be insurmountable without innovative solutions (Gerino et al., 2014). In response to this pressing need, our research undertakes an exploratory investigation into the semiotic potential (Bartolini Bussi & Mariotti, 2008) of a teaching tool widely recognized across different educational contexts throughout the world (e.g., Adelakun et al., 2025; Akbayrak & Douglas, 2021; Reddy & Sujathamalini, 2006): the Cubarithm. We have chosen it because it is paradigmatic for the initial learning of mathematics, that is emblematic of numeracy introduction in primary school for children with visual impairments. It is a tactile mathematical artifact, explicitly designed to enhance mathematical learning for children with visual impairments. In this educational landscape, the challenges extend beyond children with visual impairments, significantly impacting their educators (e.g., Rule et al., 2011). Teachers find themselves navigating complex terrain, often grappling with limited resources and a lack of professional education in using mathematical instruments

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(in the sense of Drijvers, 2019), like Cubarithm. These challenges highlight the dual nature of our research objective:

1. To uncover if and how the Cubarithm, when effectively guided by knowledgeable teachers, contributes to creating an inclusive mathematics education environment, within the classroom, for children with visual impairments.
2. To shed light on the intricate semiotic relationships within the Cubarithm that might facilitate mathematical understanding.

The purpose of this study is to provide insights about the Cubarithm as a semiotic tool, which can then inform the professional development to be provided to teachers who have to decide about the use of this resource.

Inclusive Education in Mathematics—The Case of the Italian Primary School

Inclusive education is a fundamental right for all students. Since the 1990s, the Italian primary school system has been recognised as an exemplary reference model for inclusive education policies in Europe, due to its long-standing legislative framework and its commitment to ensuring the active participation of all students in general educational settings. This development began with the enactment of Law No. 104/1992 and was subsequently reinforced by key legislative measures, including the *Guidelines for the Inclusive Education of Students with Disabilities* in 2009, Law No. 170/2010, the Directive on *Special Educational Needs* in 2012, and the *Inclusion Decree* for the years 2017–2019. The core principles underpinning this paradigm shift within primary education include the acceptance and appreciation of diversity as a source of enrichment, and the educational significance of catering to the individual needs of each student, not limited to those with specific impairments. Furthermore, the personalization and individualization of educational approaches, along with the development of inclusive pedagogical strategies, have played a crucial role in shaping a more inclusive primary education system. These initiatives extend beyond disability-related challenges, encompassing students facing socio-environmental, cultural, or familial difficulties, thereby fostering a broader culture of inclusion (cf. Italian Ministry of Education's institutional webpage on this issue: https://www.miur.gov.it/web/guest/inclusione-e-intercultura). Despite this long tradition which remains a crucial foundation and the main rationale for contextualizing our study, as is often the case, there is a significant gap between policy and practice: particularly in mathematics-related subjects. What happens is that, despite a strong and well-founded pedagogical background, and the existence of technologies (digital or analogical), educators often lack confidence in their mathematical content knowledge (Piroi et al., 2023). The special education tradition is well-established in both university instruction of educators and research, but it rarely engages in dialogue with the mathematics education community, and vice versa. Things can be arguably trickier when narrowing the field to visual impairments:

> There are no [at least Italian] text[book]s that adequately address the topic of visual impairment in a way that is accessible and usable by teachers and educators working with these students. […] The need to investigate the developmental processes of these individuals arises from the fact that they are unique and specific to the disability, and cannot be compared to normal trends in terms of the presence or absence of certain abilities or skills (Bonfigliuoli & Pinelli, 2010, p. 7, our translation).

The Unique Challenges in Mathematics Education for Pupils with Visual Impairments

> Blindness, in itself, does not seem to be an impediment to learning mathematics. Indeed, history shows that there have been a number of very successful blind mathematicians […] the lack of access to the visual field does not diminish a person's ability to visualize—but modifies it, since spatial imagination amongst those who do not see with their eyes relies on tactile and auditory activity (Healy & Fernandes, 2014, pp. 61–62).

This statement offers at least two significant starting points. Firstly, it underscores how sight can be effectively replaced by other senses to access mathematical content, and the hands emerge as the most prominent surrogate for the eyes when engaging with mathematics, mainly because such content inherently entails manipulation of spatial representations and information (Healy, 2015; Marichal et al., 2022). Similarly, Vygotsky (1993) had previously argued that replacing sight with tactile exploration through the hands can lead to the emergence of unique perspectives. These perspectives differ from those of sighted students because of the different sensory tools used to access mathematics and construct mathematical understanding. Secondly, the enduring interconnectedness of visualisation and vision within the field of mathematics education is still emphasised. Indeed, Radford (2010) notes that the learning process in mathematics, particularly within the context of algebra, is linked to a gradual "domestication *of the eye*" (p. 10, our emphasis). This involves a lengthy evolution in which individuals learn to perceive and recognise—especially with their eyes—mathematical phenomena through culturally ingrained and effective means and methods. In this regard, according to Sfard (2008), the mathematical discourse is defined by specific *visual mediators*, i.e., "visible objects that are operated upon as a part of the process of communication" (p. 133). Certainly, these quotes do not suggest that the scholarly community limits mathematics to the visual sense. Rather, the intention is to emphasise that the existing literature portrays mathematics as primarily *visual* (Arcavi, 1999).

Sight is known for its synthetic and global nature, allowing individuals to perceive the whole. This means that vision enables the simultaneous perception of multiple elements within a mathematical representation, facilitating an immediate grasp of overall structures, patterns, and spatial relationships. In contrast, touch, which is predominantly used by learners with visual impairments, relies on haptic exploration, where the whole emerges from the relationships between the parts. Learners with blindness tend to describe properties and relations using dynamic means that correspond to their physical interactions with objects (Healy & Fernandes, 2011). In particular, Braille codes, mainly characterised by a linear nature, serve as tactile substitutes for conventional written materials: the mathematical and thus spatial objects are converted into sequential representations within the Braille notation. This requires the incorporation of additional symbols (e.g., prefixes and suffixes),[1] which introduces an additional cognitive load on students and it naturally slows down their work (Edwards et al., 2006; Archambault et al., 2007). For example, a "number mark" must be placed before any number. However, this is just the simplest case: in practice, Braille mathematical notation requires numerous additional symbols, all arranged sequentially within the same line of text. Compounding this issue is the fact that Braille readers can only perceive one (and contextualised) part of the notation at a time, limiting their ability to gain a comprehensive overview of representations. To address the unique challenges that students with visual impairments face in accessing and learning mathematical content, educators have turned to multimodal, and in particular tactile, kinaesthetic and even speech-based approaches to learning (e.g., Healy & Fernandes, 2011). These methods allow students to physically interact with mathematical concepts; and tactile tools, including physical artifacts, are essential for creating a multisensory learning environment for pupils with visual impairments. However, it has only been a few years (e.g., Ahmetovic et al., 2017; Stylianidou & Nardi, 2019) since such unique perspectives due to different sensory access have been considered as an opportunity for the mathematics education of all students.

Mathematical manipulatives play a crucial role in mathematics education, serving as tangible tools that enable students to interact with mathematical concepts. Bartolini Bussi and Martignone (2014) distinguish between two primary categories (concrete and virtual) and define mathematical manipulatives as "artifacts used in mathematics education […] handled by students in order to explore, acquire, or investigate mathematical concepts or processes and to perform problem-solving activities drawing on perceptual (visual, tactile, or, more generally, sensory) evidence" (p. 365). In summary, concrete manipulatives are physical objects that

[1] This additional amount of Braille-specific symbols contributes to increase the cognitive load in reading mathematical notation in Braille. An attempt to mitigate this problem has been made in Braille codes design for the use of Braille displays. Braille codes on paper utilise six-dot cells, allowing for the representation of up to 64 symbols. In contrast, Braille displays employ cells with two additional dots, resulting in 8-dot Braille, which can encode up to 256 symbols per cell. These extra symbols help reduce the need for prefixes and suffixes, thereby minimising ambiguities, particularly in mathematical notation. As a result, most 8-dot Braille codes eliminate the use of signs for numbers, lowercase, and uppercase letters. This reduction enhances the tactile readability of mathematical expressions. Furthermore, there is not a universally accepted Braille code for mathematics (Marcone & Penteado, 2013). Efforts to standardize at least 8-dot Braille codes across various European countries have been made in the LAMBDA project (Edwards et al., 2006; Schweikhardt et al., 2006). Despite these efforts, nowadays still remain differences among Braille codes for mathematics.

students can touch, perceive, and manipulate. They provide students with a rich sensory hands-on experience and allow them to explore mathematical concepts in a tangible way. Manipulatives are frequently employed in preschool and primary education, as well as in special education settings, where they serve as essential tools for developing students' foundational mathematical understanding (e.g., Kamii et al., 2001; Simon, 2022). However, it is paramount to emphasise that their educational value extends to secondary education and broader learning contexts. The National Council of Teachers of Mathematics (NCTM) has consistently encouraged the use of manipulatives at all grade levels, dating back to 1940. This advocacy reflects the belief that manipulatives can foster essential mathematical processes such as defining, conjecturing, arguing, and proving. However, regardless of context, the autonomy of students in using manipulatives remains a subject of scrutiny. Research indicates that providing teacher guidance significantly enhances learning outcomes compared to unassisted discovery, as indicated by a synthesis of the literature on instructional guidance (Alfieri et al., 2011). The introduction of an artifact into the classroom does not inherently predetermine how students will use, engage with, perceive or interpret it. Instead, it can give rise to a range of interpretations and viewpoints among students. In essence, manipulatives, like other educational tools, carry multiple layers of meaning, with the potential to generate diverse interpretations. This characteristic aligns with the concept of *polyphony* (Bartolini Bussi et al., 2005, p. 79), which refers to the multiplicity of voices or meanings that emerge during the use of these tools in classroom interactions. Each participant to the discussion may interpret the situation differently, based on their prior knowledge, cognitive frameworks, and interaction with the manipulative. This perspective is consistent with the notion of *theoretical ambiguity* discussed by Nührenbörger and Steinbring (2008).

> This very ambiguity makes manipulatives suitable to all school levels, up to university […]. This requires a very strong and deep analysis of manipulatives, from theoretical and epistemological points of view, and a study of the consequence of this analysis in teachers' design of tasks and interventions in the mathematics classroom (Bartolini Bussi & Martignone, 2014, p. 490).

The teacher assumes a crucial role in mediating mathematical meanings through the strategic use of manipulatives as tools for *semiotic mediation* (Bartolini Bussi & Mariotti, 2008). In the absence of teacher guidance, there is a risk of disconnection between students' hands-on interaction with manipulatives and the broader mathematical content entangled in the educational intentionalities of the teacher, potentially hindering learners' construction of mathematical knowledge. Within this theoretical framework, embedded within the Theory of Semiotic Mediation (Bartolini Bussi et al., 2005), the concept of the *semiotic potential* of an artifact takes centre stage. It encompasses the dual semiotic connection that an artifact can establish. On one hand, it relates to the personal meanings that emerge from the use of the artifact to solve a specific task. At the same time, it involves the mathematical meanings evoked by the use of the artifact, which are recognisable as mathematics by an expert. This study is framed within this context and a more detailed discussion of the theoretical framework is available in the dedicated section.

The Cubarithm

Cubarithm is a specialised tactile concrete mathematical manipulative designed to facilitate the learning of basic arithmetic for individuals with visual impairments. A Cubarithm setup is shown in Fig. 1. On the left, a white box contains the digit-cubes; just below, six black digit-cubes display all the possible faces of each cube (all cubes are identical); on the right, a columnar-layout addition with an intentional mistake is represented on a white grid-slate.

There are also virtual versions of it, which can speed up the slow performance of the student using the material pieces of the artefact (Mikułowski et al., 2016), but in this study the focus is on the physical artefact.

Cubarithm consists of a set of (100) small plastic or metal cubes with an edge of 1 cm, designed to fit into a tactile slate featuring square holes. A slate is typically provided with 15 cells vertically and 20 horizontally, totalling 300 cells, with dimensions measuring approximately 1 × 19 × 25.3 cm (Aparecida & Rodrigues, 2014; Turella & Conti, 2012). However, variations in size and shape are possible, such as square slates measuring about 1 × 30 × 30 cm, or those comprising 15 × 20 cells (Fig. 1), depending on the manufacturer. These dimensions precisely match those of the cubes, ensuring easy placement and rearrangement within the grid structure, clearly demarcated for user convenience. The six-faces cubes are uniform in size and shape, featuring rounded edges and corners. The faces serve as the canvas for Braille representations of numerical values from 0 to 9. Specifically, on five

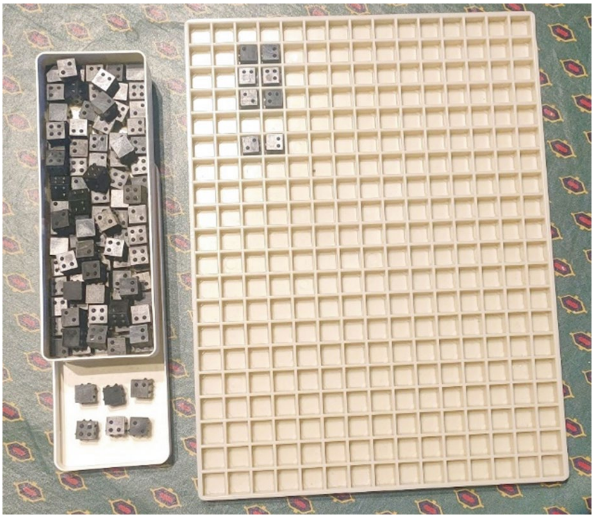


Fig. 1 A cubarithm

Fig. 2 An example of some Braille-labeled cubes (Retrieved from: https://www.braillenews.cloud/cubaritmo)

of the cube's face a single Braille sign representing digit(s) is displayed (the sixth face remains empty) The Braille numerals are created with raised dots or tiny pyramids, enabling tactile differentiation and identification by users (Fig. 2).

Cubarithm is explicitly designed to perform the four "traditional" columnar-layout operations, sometimes referred to as "operations under the line" (Mikułowski & Terlikowski, 2020, p. 290). The Cubarithm was introduced to address the limitations of using Braille on paper for performing arithmetic operations arranged in a columnar layout. There are two primary methods for writing Braille on paper: the slate and stylus, and the Braille typewriter. Writing with a slate and stylus involves using a manual tool to emboss raised dots on paper. The slate consists of two hinged plates: the top plate has small openings for each Braille cell, while the bottom has indentations to guide the stylus. A sheet of paper is placed between the plates and the stylus puns dots into the paper from right to left. Once the writing is completed, the paper is flipped over to read the Braille from left to right. This method requires performing arithmetic operations in a columnar layout from left to right while writing so that they can be read from right to left, thus creating a significant cognitive burden that is impractical for pupils learning arithmetic. This issue is overcome by using a Braille typewriter, as characters are written from left to right by pressing key combinations corresponding to the raised dots in each Braille cell. However, difficulties arise in achieving precise vertical alignment of digits. Maintaining proper column alignment requires manually repositioning the typewriter's cursor to find the correct column and adjusting the paper around the roller to ensure correct row placement. Additionally, with both the slate and stylus and the Braille typewriter, erasing individual characters and replacing them with correct ones without damaging the paper is extremely difficult. With the Cubarithm, the digit-cubes are positioned and removed from the various horizontal and vertical cells. In Fig. 1, the slate displays an example of an incorrect columnar-layout addition (12 + 34 + 56 = 92). The top row and the first two columns from the left are empty. The representation of the addition "under the line" with three addends then begins. The first row contains the digit-cube 2 on the right and the digit-cube 1 on the left. The second row has the digit-cube 4 on the right and the digit-cube 3 on the left. The third row features the digit-cube 6 on the right and the digit-cube 5 on the left. An empty row marks the position of "the line" separating the addends from the result, which is placed below. Here, the digit-cube 2 is on the right and the digit-cube 9 on the left. For reference, Fig. 3 presents the analogous columnar-layout addition on paper, demonstrating how this notation is transposed onto the Cubarithm slate in Fig. 1.

Fig. 3 The incorrect 12 + 34 + 56 = 92 columnar-layout addition on grid paper

Theoretical Framework

As stated in the Introduction and to address the research aim, the necessary theoretical framework comprises the theoretical concept of semiotic potential, contextualised within the framework of Inclusive Mathematics as proposed by Moura (2020). This combination of frameworks allows for the analysis of whether Cubarithm can be, and how it can be, an inclusive "mediator" of educational interaction patterns and a meaningful producer of mathematical signs.

The Semiotic Potential

The Theory of Semiotic Mediation is grounded in a Vygotskian perspective on the teaching-learning of mathematics. Here, Bartolini Bussi and Mariotti (2008) intend learning as a collective process of the class group, concerning the evolution of signs used by students to discuss the mathematical meanings. In particular, the two researchers propose to structure teaching activities according to repeated iterations of the so-called didactic cycle: students are first asked to solve a task using a specific artifact; then, on the basis of artifact signs (or situated signs) emerged during that experience (i.e., informal signs, such as words and gestures, used by the students to talk about what they have experienced with the artifact), the teacher can orchestrate a phase of mathematical discussion (Bartolini Bussi, 1998). This phase is aimed at making these artifact signs evolve towards targeted mathematical signs (i.e., the signs of formal mathematics referring to the relevant mathematical meanings, that the teacher aims for students to co-construct).

For the design of a didactic cycle, a particularly important role is played by the teacher's choice of the artifact and the task for students to accomplish, in relation to the mathematical knowledge to be taught. Those choices, in fact, affect the emergence of situated signs suitable to unfold significant mathematical meanings. The artifact is conceived as a historical-cultural product, created to simplify or to solve a particularly frequent problem in a certain historical and cultural period. The artifacts of interest in mathematics usually work on the basis of culturally and historically determined parts of mathematical knowledge. For example, the compass incorporates the meaning of a circle as the locus of points equidistant from a center; the abacus incorporates different ways of writing, composing or decomposing numbers depending on their cultural origin (see Bartolini Bussi et al., 2018; Maschietto & Bartolini Bussi, 2011). The mathematical knowledge "incorporated" into the artifact's functioning, according to Bartolini Bussi and Mariotti (2008), is what makes it possible for situated signs to evolve towards targeted mathematical ones. As Baccaglini-Frank et al. (2023) explain

> artifacts afford a *semiotic potential* (Bartolini Bussi & Mariotti, 2008): the use of an artifact for accomplishing a task fosters the emergence and transformation of signs (that is verbal utterances, written inscriptions, gestures, …) related to the activity at stake (called situated signs); in turn, an expert can relate such situated signs to the appropriate mathematical knowledge. The *semiotic potential of an artifact with respect to a task and to certain mathematical meanings* is precisely its potential to foster the emergence of situated signs that are, on the one hand, related to the accomplishment of the task, and, on the other hand, relatable to mathematical meanings targeted by the teacher (p. 4).

Thus, the capacity to discern the semiotic potential of an artifact (with respect to a task and to certain mathematical meanings) is what allows the teacher to predict and manage the evolution of situated signs towards the targeted mathematical ones:

> any artefact will be referred to as tool of semiotic mediation as long as it is (or it is conceived to be) intentionally used by the teacher to mediate a mathematical content through a designed didactical intervention. […] The key point according to our hypothesis is that the twofold role played by the artefact, both as a means in accomplishing a task, and as a tool of semiotic mediation to accomplish a didactical objective, can be fully exploited. The role of the teacher is then crucial and not incidental and the teaching sequence has to present certain peculiarities [referring to the structure of the didactical cycle] (Bartolini Bussi & Mariotti, 2008, p.754).

The Inclusive Interaction Pattern

This study is in alignment with Moura's (2020) and Skovsmose's proposals of engaging all the learners in the existing curriculum (in Figueiras et al., 2016, pp. 19–20), "learning in mixed-ability groups provides richer opportunities for learning, as learning is related to processes of negotiating, explaining, and noticing". However, the use of assistive technologies does not guarantee the inclusion of students with visual impairments in teaching-learning practices (Ahmed & Chao, 2018). In our vision, "the notion of inclusion goes beyond simple, though preliminary, accessibility. […] [T]he presence of these technologies in classrooms, where students with and without visual impairments are together, is the first step in promoting a new classroom relationship and communication" (Piroi et al., 2023, p. 60).

In mathematics classrooms, interactions between students, and even between the class teacher and the student with visual impairments, are frequently indirect. Communication and engagement typically occur through the intermediary role of a support teacher specializing in special educational needs. Here, prevailing pedagogical approaches often substitute visual cues with alternative sensory modalities, thereby expecting students with visual impairments to conform to the behaviours of their sighted peers (Ahmed & Chao, 2018). This trend is illustrated in Fig. 4.

In an inclusive classroom, with the presence of assistive technologies as inclusive instruments, this pattern evolves into the one represented in Fig. 5, as demonstrated by Piroi et al. (2023). For instance, consider a mathematics activity in which students explore numeric patterns and sequences. In a traditional setting (Fig. 4), a student with visual impairments might rely on a support teacher to describe what is being drawn or written by their peer, creating an indirect mode of interaction. However, with swell paper or a digital board equipped with screen-reading software and a tactile display, the student can directly access and manipulate the different representations. For example, if the class is diagrammatically identifying the rule behind a sequence, the student can explore the patterns using audio feedback and tactile overlays, actively participating in the discussion rather than receiving second-hand explanations. This transformation allows all students to engage directly with mathematical structures, as shown in Fig. 5, fostering more direct and inclusive interactions. Tactile tools are essential for creating a multisensory learning environment with inclusive interactions for students with visual impairments. However, this connection between tactile perception and visual impairment often results in accommodating students with visual impairments by modifying visual materials typically designed for sighted students. While some accommodations have been successful, such as creating tactile shapes for students with visual impairments while the rest of the class uses visual shapes from the standard textbook (Argyropoulos & Stamouli, 2006), other accommodations face various limitations. These limitations can be

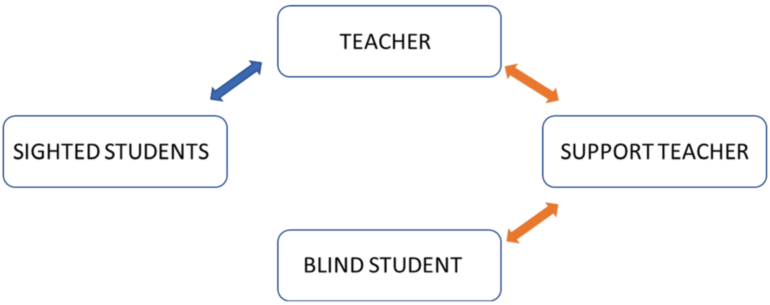


Fig. 4 Interaction patterns in classes with blind (sic.) students and a support teacher (Piroi et al., 2023, p. 61)

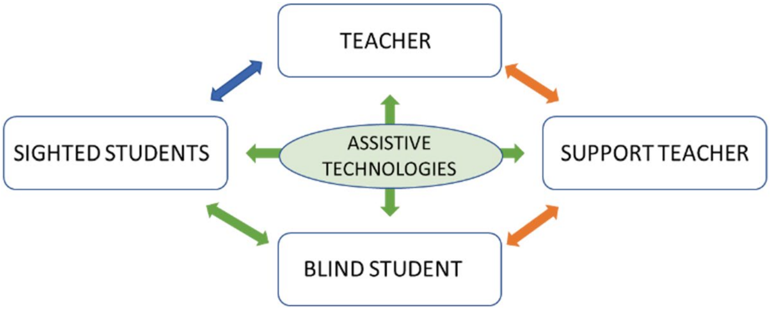


Fig. 5 Inclusive interaction patterns, mediated by assistive technologies (Piroi et al., 2023, p. 72)

categorised as technical, affective, or social. In this study, an analysis of the semiotic potential of the Cubarithm is conducted to examine how these three limitations could be related to semiotic mediation processes and the emergence of inclusive interaction patterns. By exploring these relationships, this study aims to provide deeper insights into how the Cubarithm can shape both individual meaning-making and classroom dynamics.

Cubarithm: An In-Depth Semiotic Analysis

This section presents an a priori semiotic analysis of the Cubarithm, aimed at investigating its semiotic potential to mediate mathematical meaning through haptic interaction. While this analysis is theoretically grounded, it emerges from a structured didactic observations (Manolino, 2024) of four young users (grades 1 and 2, with different visual impairments) engaged in simple tasks on columnar-layout calculations designed by their classroom teachers, as well as an interview with an adult blind user, expert in STEM, who had extensive experience interacting with the Cubarithm during his primary education. These empirical insights have informed our understanding of the challenges and affordances associated with this artifact. However, to our knowledge, no task-based studies explicitly informed by the framework of Semiotic Mediation have yet been conducted or documented in the literature. The absence of such research underscores the need for future empirical investigations that systematically explore how learners construct mathematical meaning through engagement with the Cubarithm in structured instructional settings. This study, therefore, serves as a preliminary step in theorizing the semiotic potential of the Cubarithm, providing a foundation for future research that integrates classroom-based interventions and controlled experimental designs. The unique design of the Braille systems used in the Cubarithm allows for a single cube to represent different digits based on its spatial orientation and for the same positioned cube to represent some other digits based on its orientation within the slate. For instance, the Braille character corresponding to the letter *b* (Braille dots 1 and 2), which represents the digit 2, can be rotated by 90 degrees to become the letter *c*

(Braille dots 1 and 4), which represents the digit 3 (see Fig. 6). Through this mechanism, each cube can be oriented and placed on the slate to assume any desired digit from 0 to 9. Figure 7 illustrates the symmetrical pairs of digits that emerge from this system.

Students can arrange cubes in various configurations within the slate, either placing them side by side vertically or horizontally to create different combinations of Braille digits.

This manipulation demands a significant amount of time and imposes a high cognitive effort, particularly for younger students in primary or nursery education. Several cognitive challenges arise:

- Symbol Recognition—At this stage of schooling, students are still in the process of learning Braille, so symbol recognition is not yet automatic.
- Orientation Awareness—Identifying the correct digit requires a careful consideration of the cube's orientation.
- Hand-Cube-Slate Coordination—The student must precisely place the cube in the correct cell while ensuring the proper orientation.
- Memory Load—Since students with visual impairments lack a synthetic and global visual perception of the slate, they must rely on memory to track which cubes have already been placed. Otherwise, they would need to frequently re-explore the slate through touch.

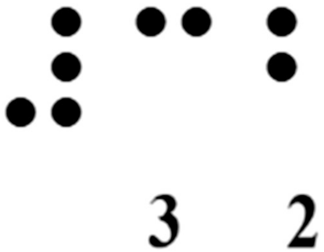
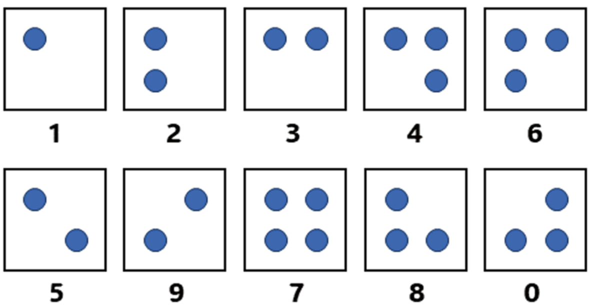


Fig. 6 A braille representation of the number 32. [Note: The first symbol is known as the number mark, i.e., a sign placed in front of digits to distinguish it from the analogous group of letters. When the number mark is omitted, this notation represents the word 'bc'.]

Fig. 7 The digits in the Braille-labeled cubes system

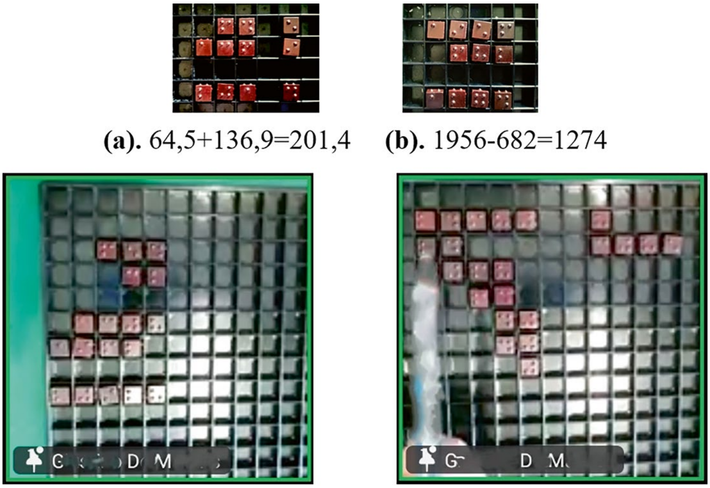


The time required for this process and the associated cognitive load are non-negligible. By contrast, in its paper-based "traditional" columnar-layout operations counterpart, columnar diagrams are designed specifically to relieve the student from this effort and to alleviate this cognitive burden. The structure of these diagrams is designed to enable a quicker and more intuitive digit placement process, allowing students to focus their cognitive resources on the calculation itself rather than on managing the spatial organization of digits.

Across various bibliographic references studied and presented so far, as well as in teacher education courses conducted (at least) in Italy to date by academic institutions and associations for visual impairments, the Cubarithm has been consistently presented as a tool for mathematics education (Del Campo, 2000). Frequently cited as either the sole or primary mathematical artifact for students with visual impairments in primary education, its use appears indispensable when addressing mathematics education in this context. However, it is typically introduced solely for computational purposes, being described as "a tool for learning traditional columnar layout calculations by visually impaired students" (Mikułowski et al., 2016, p. 17). The Cubarithm is thus regarded as a faithful physical counterpart of what is commonly written on a paper sheet in mathematics lessons with squares in its entirety. Unlike the Braille slate and Braille typewriter, it enables the precise spatial placement and removal of digits within a defined grid structure, making them easy to locate through touch. Consequently, a student with visual impairments can perform traditional columnar layout calculations in a way that is considered analogous to how sighted peers work with pen and paper, although the situated signs will be predominantly haptic rather than graphic-script type. However, it is crucial to recognize that the teaching of "operations under the line", if not strongly anchored in their meaning, leads (all) students to rely on more or less automated procedures for spatial displacements of digits. As described by Orton-Flynn and Richards (2000) in their discussion of dyslexics—a perspective that can be extended to learners more broadly:

> [students] can be confused about the direction in which they should work on a particular calculation. Western languages are read and written from the top left to the bottom right of the page. For multiplication, however, pupils are taught to work out from the bottom right, diagonally across to the top left. Addition is done by calculating from the top of the furthest right-hand column, and working towards any left-hand columns. Division is, again, laid out quite differently. Is it so surprising that many pupils become confused? (p. 206).

It is even more complicated for a student with visual impairments who cannot grasp the whole view. Figure 8 provides four representations of operations on the cubarithm. It is evident how close the transposition from the paper is. But how to keep track of the intermediate steps? In a faithful transposition, there should also be traceability of intermediate signs, such as remainder and decomposition signs. How can this be done in the cubarithm without confusion? Some educators place such signs above the first row of the operation, but in this way a re-reading in case of forgetfulness, how does it take into account what was already there and what was added afterwards during the calculation?



(a). 64,5+136,9=201,4 **(b).** 1956-682=1274

(c). 654x32 **(d).** 24680:8=3085

Fig. 8 Four "operations under the line" displayed during a teacher education course

Despite this, in teacher education courses, Cubarithm is almost always taken to the extent of providing rules for carrying out column operations, both with and without decimal numbers. A commonly found description is that, using the Cubarithm, arithmetic operation signs (+, −, ×, ÷, =) are generally omitted. These symbols are expected to be inferred from the context, as their inclusion is thought to lead to confusion. This arises from the fact that the Braille representations of digits within the Cubarithm are encoded using four dots, making some operation symbols visually and tactilely similar to certain numbers (e.g., '+' resembling the digit 6, ':' resembling 4, '×' resembling 8, and '='resembling 7). Cubarithm, like Braille, is thus heavily context-dependent, relying on implicit conventions for mathematical expressions.

Another characteristic aspect of this artifact is its ephemeral nature. A frequently cited analogy in teacher education materials describes it as "a 'rough draft' paper notebook with squares: once the exercise is completed, all cubes are removed and stored back in the designated container"—as literally quoted from a recent Italian teacher education course handout. This contrasts with traditional paper-based work, where the permanence of written steps serves an essential educational function. Despite the Cubarithm intention to replicate the use of a sheet of paper, it does not retain all its aspects. In fact, at the educational level, one of the most essential aspects is lost: the permanence of errors and processes. The absence of a record of previous steps poses a challenge: the learning process benefits from the ability to review past work, identify errors, and reflect on the sequence of calculations. The Cubarithm, in its standard use, does not support this type of reflective engagement. In addition to the effort and slowness of manipulating and positioning the cubes on the slate, the transcription of what happens on the Cubarithm cannot occur. And if writing on paper or digital devices is possible, in any case for students with visual impairments it is linear: the spatial dimension of the columns is lost.

Despite these limitations, the Cubarithm can serve valuable preparatory functions in early mathematics education. For instance, its interactional properties differentiate it significantly from standard paper-based mathematical notation. It can be used in activities designed to stimulate cognitive skills such as spatial alignment, laterality, and orientation tasks. Additionally, it can be employed to reinforce mathematical language and Braille notation through structured exercises (e.g., RoboBraille.org, 2016). The design of the slate features a smooth, flat surface that offers stability during mathematical interactions, providing ample space for arranging and organising the Braille-labeled cubes in the space. The grid structure provides a tactile framework for arranging haptic elements, allowing users to create modelizations and mathematical structures, and explore mathematical relationships through touch. From a semiotic perspective, the slate serves as a physical medium for a tactile sign system. Its grid structure provides a haptic framework for arranging mathematical signs, allowing the user to construct and explore through touch. This physical arrangement introduces a structured spatial representation and reasoning that can support early algebraic reasoning. The process of manipulating the cubes involves a continuous interplay between perception, action, and cognition, reinforcing the semiotic mediation between the physical manipulative and the abstract mathematical concepts it represents. The need to physically rotate and place the cubes correctly could mirror essential mathematical processes such as rearranging terms, aligning place values, and structuring relations, making it a tool that may extend beyond mere computation. At the same time, the Cubartithm requires careful instructional strategies to mitigate cognitive overload, ensuring that students can focus on conceptual understanding rather than on the mechanics of manipulation alone. Its use in mathematics education should be carefully contextualized. Rather than serving solely as a computation aid, it may be more effectively integrated into broader pedagogical strategies that incorporate multiple modalities, including digital and paper-based tools, to support a more comprehensive mathematical learning experience.

Discussion and Conclusions

The semiotic analysis conducted in this study has revealed how the Cubarithm encapsulates the technical, affective, and social limitations commonly encountered in adapting tactile mathematical tools originally designed for sighted learners. These limitations, which influence both the usability and educational effectiveness of the tool, must be considered in relation to how the mathematical meaning is constructed through interaction with the artifact. As established in the theoretical framework, an artifact's semiotic potential is crucial in determining how it can foster the emergence of mathematical signs and contribute to semiotic mediation in the classroom. The Cubarithm, while historically recognized as an essential computational tool for students with visual impairments, presents challenges that can directly impact both the evolution of mathematical meaning and the inclusivity of interaction patterns in the learning environment.

The technical limitations of the Cubarithm concern the potential motor and spatial-perceptual difficulties that students may encounter when manipulating the small cubes (for instance, while correctly positioning them on the slate). These tasks require fine motor skills and spatial reasoning, which can be especially demanding for young learners or those still developing proficiency in Braille, and which Cubarithm does not support, but rather tires them out. Additionally, the lack of a permanent external representation of their work, particularly concerning errors, prevents students from easily reviewing and reflecting on their processes. These technical limitations may, in turn, lead to educative and affective constraints. The cognitive burden associated with handling the Cubarithm may shift students' focus away from conceptual engagement and towards the mere mechanics of digit placement. This misalignment between effort and learning outcomes can diminish students' perceived self-efficacy, potentially reducing their motivation to engage with mathematical tasks. Specifically, the slowness and challenges in placing and identifying the cubes on the Cubarithm impose a significant workload, diverting attention from the primary objective, which is to fulfil the specific mathematical task. Consequently, there is a risk that students with visual impairments may perceive a decrease in autonomy and thus self-efficacy, resulting in decreased interest in mathematics (Bernabè, 2024; Mason & McCall, 2013). Moreover, the social limitations of the Cubarithm arise from its reliance on Braille-labeled cubes, which are inaccessible to peers and teachers unfamiliar with Braille. This characteristic may result in reduced interaction between the student with visual impairments and their peers, often resulting in the isolation of the student with visual impairments rather than fostering inclusion and multimodality. As demonstrated in the analysis, the Cubarithm's role as a mediator of inclusive interaction patterns is compromised by its reliance on a not shared socio-cultural language and sign production. It does not fully facilitate shared engagement in mathematical discourse. Here the Cubarithm fails in its assistive and inclusive role (see Fig. 5). Currently, alternatives to the Cubarithm based on digital devices could render this tool more inclusive. However, the skills required to use such tools may not align with those typically expected in primary education. Young students may lack proficiency in 10-finger typing, which is necessary for efficient input on digital devices, and may also require extensive training to develop a comprehensive understanding of Braille on refreshable Braille displays. Additionally, the transition from a tangible, spatially structured artifact like the Cubarithm to a linear, text-based digital format may introduce further cognitive challenges, particularly in early mathematical learning, where direct manipulation of objects plays a crucial role in conceptual development.

Despite these limitations, the Cubarithm remains a particularly useful tool in teaching students with blindness, provided that its use is expanded beyond simple traditional use for teaching column-based computational tasks. The Cubarithm offers numerous opportunities for educational applications. Firstly, the ability to directly manipulate Braille-numbered cubes in a tangible two-dimensional space allows for the design of educational activities aimed at developing visuospatial and visuoconstructive skills, such as Euclidean plane isometry (see the issue of cubes rotation and slate spatialisation) or visual pattern proposals. More generally, the Cubarithm can serve as an effective semiotic mediator if integrated into activities designed to enhance spatial reasoning, lateralization, and proprioception. The development of spatial reasoning and visuospatial skills is a fundamental aspect of early education, as these abilities support a wide range of cognitive and literacy-related processes. In sighted learners, the coordination between visual perception and motor actions—commonly referred to as oculomotor coordination—plays a crucial role in integrating spatial information (Pento, 2020), facilitating the discrimination of similar signs, and supporting the structured organization of written language and representation (Colina, 2015). Moreover, successful encoding relies not only on semantic acquisition but also on the ability to analyze and process spatial control and visuospatial characteristics, skills that progressively develop throughout early childhood (Cornoldi et al., 1997). For students with visual impairments, the absence of direct visual input necessitates alternative pathways for acquiring these essential competencies. The Cubarithm, by enabling direct manipulation of signs within a structured two-dimensional space, provides a haptic analogous to the role that written text and diagrammatic representations play in sighted learners' development (Olive & Passerault, 2012; Tseng & Cermak, 1993). Through targeted educational activities, this tool can support the refinement of spatial reasoning, promote intermodal integration, and foster a deeper understanding of mathematical structures. In this sense, the Cubarithm slate can serve as a physical counterpart to the sheet of paper, a conventional artifact essential for spatial control, organization, and structuring in early education. Additionally, it can be employed to develop the skills necessary for transitioning from a tangible space to a virtual one, accessible through tools like keyboards, voice feedback, or Braille displays. Individuals with blindness face difficulties in mastering virtual spaces without adequate experience in physical space through object manipulation activities (Del Campo, 2000), for which the Cubarithm may be particularly well-suited. The Cubarithm can facilitate the development of primary school-level mathematical skills, such as completing numerical sequences, and secondary school-level skills, such as introducing coordinate systems, the concept of matrices, or algebra structure sense (Bernabè, 2024; Maffia et al., 2025).

Indeed, the Cubarithm can regain its role as a mediator for inclusive interaction patterns in the classroom if it is used for educational purposes that extend beyond calculating in columns, and rather work on spatiality, lateralisation, sheet space usage and proprioception. In this case, not only does the Cubarithm become an essential tool for the student with visual impairments, but it acts as a pivotal tool for the whole class to develop these target competencies for the early years of primary school. In the context of elementary education, learning mathematics involves more than performing arithmetic calculations. It requires understanding numerical structures, recognizing patterns, and developing problem-solving skills. The Cubarithm, when used solely as a computational tool, does not inherently support these broader learning objectives. Furthermore, educational research suggests that multi-sensory learning environments enhance concept retention and comprehension. Combining the Cubarithm with auditory feedback systems or tactile-graphical representations could mitigate some of its limitations and expand its role beyond arithmetic operations, fostering a deeper engagement with mathematical concepts.

Often mathematics education, as challenging for teachers caring for students with special needs, is conveyed through a "replication" of tools and procedures typical of traditional teaching-learning, instead of providing an opportunity to realise an inclusive mathematics teaching-learning. We look at Inclusive Mathematics as a framework to renew the conceptualisation of mathematical contents through a multimodal and sensorial interaction pattern.

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