# ANALYSING THE TRANSITION BETWEEN PERCEPTION–ACTION LOOPS IN BLIND LEARNERS' AUDIO–MEDIATED MATHEMATICAL ACTIVITY

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*This study investigates how a blind learner engages with function graphs through* Audiofunctions, *an audio–based digital tool. Adopting the Body–Artifact Functional System (BAFS) as a theoretical framework, this contribution focuses on the transition from perception–action Loop 3, in which the artifact remains an external object of attention, to Loop 4, where it becomes functionally integrated into perception. Using a qualitative case study, a micro–episode from a task–based interview is analysed, revealing how auditory feedback shifts from being an object of exploration to a perceptually transparent means of accessing mathematical structure. These findings contribute to the development of BAFS in non–visual contexts and highlight the epistemic role of audio–based tools in inclusive mathematics education.*

## INTRODUCTION

Mathematics education has traditionally relied on vision. Mathematical objects, such as functions, are commonly introduced and explored through visual representations such as symbolic expressions, diagrams, graphs, and spatial layouts. While these representations support mathematical reasoning for sighted learners, they also constitute barriers for blind learners, whose access to mathematical structures depends on alternative perceptual resources, such as auditory, tactile, and proprioceptive modalities, often mediated by digital tools.

In response to these challenges, research in mathematics education has increasingly acknowledged the embodied nature of mathematical activity, emphasizing the role of perception, bodily action, and interaction with artifacts in the construction of mathematical meaning. Within this perspective, the Body–Artifact Functional System (BAFS) (Shvarts et al., 2021) conceptualizes mathematical activity as a dynamic system organized through perception–action loops between the organism's body potentialities, the artifact affordances, and the affordances of the surrounding environment, driven by multilevel intentionality.

Despite its theoretical potential, BAFS–based research has been conducted almost exclusively with sighted learners in visually mediated digital environments, frequently relying on eye–tracking as a primary analytical resource. As a result, vision is implicitly treated as the dominant perceptual modality, and little is known about how



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perception–action loops reorganize when visual access is reduced or absent. In particular, the transition from engaging *with* an artifact as an external object to perceiving and acting *through* it as an embodied extension of their perceptual system remains underexplored for visual impairments contexts.

For blind learners, audio–based digital environments play a central role in mediating mathematical activity. Rather than approaching auditory perception as compensatory or assistive substitute for vision, this study treats sound as a constitutive component of embodied mathematical activity. We examine how a blind learner engages with function graphs through *Audiofunctions*, an audio-based tool developed by the "S. Polin" Laboratory in Turin, that affords the sonification of functions in the Cartesian plane (Manolino et al., 2025). The guiding research question is: *How does the transition from perception–action Loop 3 to Loop 4 emerge in a blind learner's mathematical activity mediated by* Audiofunctions*?*

## LITERATURE REVIEW

Research in mathematics education has increasingly drawn on embodied and ecological perspectives to account for the role of perception, action, and tool use in mathematical activity (Nemirovsky & Ferrara, 2009; Radford, 2014). From this standpoint, digital tools are understood as active participants in mathematical activity, shaping how learners perceive and act on mathematical objects. Gibson's (1986) notion of affordances highlights the inseparability of perception and action, and has informed research on how tools can become functionally integrated into the body during purposeful activity.

Within accessibility research, studies have shown that blind learners engage with mathematical concepts through perceptual modalities other than vision, most notably auditory and tactile–haptic systems (e.g., Manolino et al., 2025; Pissaloux et al., 2026). In this context, *sonification* has been explored as a way of representing mathematical structures through sound, allowing learners to access information sequentially and dynamically, rather than through static visual displays (Hermann et al., 2011). These approaches suggest that sound can serve not merely as an assistive aid, but as a meaningful perceptual channel for mathematical exploration. However, how such forms of mediation reorganize perception–action loops within embodied theoretical frameworks remains largely unexplored. The present study addresses this gap by examining audio-mediated mathematical activity through the lens of BAFS.

## THEORETICAL FRAMEWORK

This study adopts the Body–Artifact Functional System (BAFS), developed by Shvarts et al. (2021), as its theoretical framework. with a specific focus on the transition between perception–action loops. BAFS conceptualizes mathematical activity as a dynamic functional system organized through perception–action loops connecting the organism's body potentialities, the artifact affordances, and the affordances of the environment. This framework is grounded in ecological approaches to perception and action (Gibson, 1986), further developed through the integration of perception and action within a unified cycle (Turvey, 2007).

Within BAFS (Figure 1), four perception–action loops are distinguished. Loop 1 concerns the relation between the body and the environment; Loop 2 connects the artifact with the environment; Loop 3 involves coordination between the body and the artifact; Loop 4 emerges when the body and the artifact become functionally integrated, operating as a single perceptual unit that perceives and acts directly on the environment (Figure 2). The transition from Loop 3 to Loop 4 marks a qualitative reorganization of activity: the artifact ceases to be an external object and becomes an extension of the organism's body and mind.

Figure 1: Body–Artifact Functional System (BAFS) in interaction with the environment (Shvarts et al., 2021)

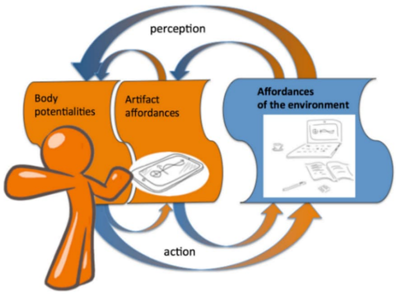
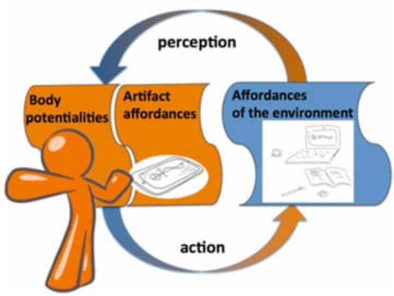



Figure 2: Instrumental genesis from a radical embodied perspective (Shvarts et al., 2021)




For blind learners, this transition is particularly significant, as access to graphical representations occurs through sequential auditory or tactile modalities rather than through vision. Engaging with audio-mediated environments requires learners to not only rely on alternative perceptual systems but to develop new forms of perceptual and cognitive coupling with artifacts. From a BAFS perspective, this coupling can be analyzed as the emergence of a body–artifact functional system in which sound functions as a perceptual medium for accessing mathematical structure. In this study, BAFS provides an analytic lens for examining how such coupling becomes embodied during audio-mediated exploration of function graphs.

## METHODS

### Participants and Design

The study adopts a qualitative case study design focusing on one blind participant engaged in mathematical tasks involving function graphs mediated by *Audiofunctions*. This case is drawn from a broader study involving ten blind volunteers; only one is presented here due to space constraints and is referred to as *Case X*. The participant was selected for his expertise in mathematical problem solving. This choice was made to limit the influence of conceptual difficulties or tool familiarization, allowing the analysis to focus on learner–artifact interaction. Data consist of screen and audio recordings of task-based interviews, which were subsequently transcribed. Case X accesses mathematical content using a Braille display and the NVDA screen reader.

## The Digital Environment

*Audiofunctions* is a free online software that affords the sonification of function graphs represented in the Cartesian plane. When audio feedback is activated, navigation along the graph is controlled through directional keys, or the entire graph can be sonified from left to right by pressing the B key. Users can switch between functions or directly access a specific function through numerical commands. The N key is used to switch to the next function, while to display a specific function immediately, the 1, 2, … keys are used, according to the function number. Two sonification modes are available: the clarinet is a continuous sonification, while the guitar is by nature a discrete sonification.

## Task Structure

The interview protocol consists of four parts. Throughout the interview, the participant is asked to verbalize observations and reasoning, describe performed actions, and explain how interaction with the artifact supports task execution.

The first part is introductory and familiarizes the participant with the software. During this phase, six functions are presented (http://tinyurl.com/testingpart0). The participant is first asked to identify salient sound features and hypothesize their correspondence with graphical properties. In a second step, the participant switches between the two sonification modes and describes which aspects of the functions become perceptually salient through each sound.

The remaining three parts consist of comparative tasks (http://tinyurl.com/testingpart1; http://tinyurl.com/testingpart2; http://tinyurl.com/testingpart3). In each task, the participant listens to pairs of function graphs, identifies similarities and differences, and explains the strategies used to support comparison. The interview concludes with a reflective question aimed at eliciting sound-based descriptions of mathematical properties emerging from the comparisons.

## Analytical Approach

The analysis focuses on the transition between two qualitatively different organizations of perception–action loops in audio-mediated mathematical activity. Specifically, it examines the shift from perception–action Loop 3, in which the artifact is treated as an external object of coordination, to Loop 4, in which the artifact becomes functionally integrated into the learner's perceptual system. From this perspective, the transition from perception–action Loop 3 to Loop 4 can be interpreted as a reconfiguration of instrumental genesis (Rabardel, 1995). While instrumental genesis traditionally emphasizes the development of utilization schemes and object mediation, BAFS reframes this process as an embodied coupling in which the artifact becomes part of the perceptual system itself. In this study, instrumental genesis is not analysed as a cognitive process in itself, but as a theoretical reference for identifying qualitative shifts in the organization of perception–action loops. The BAFS framework guides the analysis by foregrounding bodily experience and tool-mediated perception in mathematical activity. Rather than centering on internal mental schemes, the analysis attends to how coordination between body and artifact stabilizes over time. Loop 3 is characterized by epistemic and pragmatic engagement with the artifact itself, as the learner explores and adjusts its use. Loop 4 emerges when the artifact becomes perceptually transparent and action is directed toward the mathematical task rather than the tool. At this stage, auditory feedback is no longer treated as information about the artifact but is directly perceived as information about the mathematical structure.

## DATA ANALYSIS

For this study, due to the limitations of the requirements for the submission, we focus on a micro–episode from the interview. This micro–episode has to do with the first part, the introductory one. The three analytical markers help to understand the passage from one loop to another: Loop 3 (artifact still external); Transition Marker; Loop 4 (the artifact becomes embodied). They are used to trace this reconfiguration of instrumental genesis across verbalizations and actions within the micro-episode.

Table 1: Analytical markers of Loop3-Loop4 transition in audio-mediated micro-episode.

| **What is said** | **What is done** | **Analytical marker** |
|---|---|---|
| ***This particular function, which I note throughout the entire exploration, throughout the sonification, basically maintains the same frequency***. And there are... | | Loop 3<br>The participant mentions explicitly the feature of sonification, such as frequency. There is an epistemic approach, based on the fact that he is exploring the sound. |
| It has the same frequency. And then there are two or three... I'll listen to them again. | The red dot moves from left to right of function 1 with the sonification of the clarinet. | |
| Okay. ***There are three elements that are highlighted: at the beginning, in the middle, and at the end.*** And then the stereophony basically moves from left to right, so to speak, ***it uses the stereophony to move from left to right.*** Then... | NVDA reads "function 2" when Case X switches to function 2 and, with B key, he activates the movement of the red dot from left to right with clarinet mode. | Loop 3<br>The attention is on the artifact, especially on the sounds he produces, such as stereophony. |

| | | |
|---|---|---|
| Okay. This function basically has the characteristic of having the same frequency, in the sense of being constant throughout the graph, but it is lower than the first. Then I have the perception... Here too, there are three points... three elements highlighted, but I have the perception that there is also a sort of very slight ***background noise***. | | Loop 3<br>He mentions the background noise. |
| Oh, okay, so yes, that background noise is clearer. ***In fact, in this case, there's basically sound; the frequency of the sound increases, and about halfway through, the noise, the white noise underneath, disappears.*** Now I'll listen to it again. | Activate the movement of the red dot along function 3 for a second time with clarinet mode. | Transition marker<br>The reference to the tool is reduced. He identifies the sound characteristics as structural indicators. |
| ***Shortly afterwards, it disappears, shortly after it practically hits the second point of interest, so to speak (*).*** And then... ***Okay, anyway, yes, this one increases in frequency (**).*** | Listen again to the movement of the red dot from right to left, then from left to right, but more slowly, still with clarinet mode. | (*) Transition marker<br>The sound is no longer an object of listening, but a way of orienting.<br>(**) Loop 4 |
| ***I'm exploring with the SHIFT now.*** | He goes back slightly to point (1;0), then, he moves the red dot to the beginning (left part of the screen), in clarinet mode. | Loop 4<br>The attention is no longer on the tool's sound but on the type of function. |
| So, wait, it was the fourth, it was yes, the third, now I'll do the fourth. | Movement of the red dot from left to right of function 4 with clarinet sonification. | |
| ***Okay, this represents the exact opposite of the third***, that is, ***it starts from a high frequency, descends, at a certain point the background noise begins***, okay? And ***with the three points of interest, let's say the three key points*** and… Yes, what else, practically the exact opposite of the third, in short. I also wanted to understand if the... | NVDA reads "function 3" and Case X goes back to function 3. He listens again to the movement of the red dot from left to right with the clarinet mode. Then he goes back to the fourth function and listens to it again, always from left to right with clarinet mode. | Loop 4<br>He can compare functions without explicitly referring to the tool. The tool is transparent now. |
| Eh, the frequency perhaps starts slightly higher than the third, but the range... is more or less the same from what... | NVDA reads "function 5" when Case X goes back to function 5. He listens to the movement of the red dot from left to right with clarinet mode. | |

## RESULTS

During the initial stage of interaction, the participant's verbalizations indicate that the artifact remains an external object. Initially, he focuses on artifact exploration, understanding how to navigate it and how it would later be useful for task resolution. Case X descriptions focus on perceptual properties of sonification, such as pitch stability, stereophony, and background noise (e.g., "it maintains the same frequency"; "it uses stereophony to move from left to right"; "background noise"). These comments characterize an epistemic and pragmatic intentionality of engagement with the artifact, in which the learner actively perceives, interprets and tests through re–exploration and re–listening its intended functions in order to understand how the tool can mediate access to the graphical representations.

The transition becomes visible when the participant begins to relate auditory features to the structure of the function or relevant points of it. For example, he notices that the background noise disappears "shortly after encountering the second point". At this stage, the sonification is transformed into a perceptual reference for navigating the function structure, i.e. the mathematical object itself. This moment marks the stabilization of body–artifact coordination.

Then Loop 4 emerges, when the attention shifts away from the properties of the artifact toward direct comparison between functions. Based on the sound, he recognize that function 3 is the opposite of function 2, noticing the difference in frequency, and the same points of interest, to which he refers at the beginning. This indicates that he is acting *through* sonification rather than on it. From the perspective of instrumental genesis, this shift corresponds to a reorientation of intentionality from the artifact toward the mathematical task, consistent with the emergence of Loop 4. The artifact becomes perceptually transparent, supporting immediate sensorimotor access to mathematical relationships. At this point, the body–artifact system operates as a unified functional unit, enabling the learner to perceive and act upon the task structure directly.

## CONCLUSIONS

This study examined the transition between two perception–action loops in a blind learner's mathematical activity mediated by an audio–based digital tool. The analysis highlights how an artifact can shift from being an explicit object of attention, external to the body, to perceptually transparent and functionally integrated into the learner's activity. The findings indicate that this transition unfolds gradually through the stabilization of sensorimotor coordination. Initially, the learner engages in epistemic and pragmatic exploration of the artifact's properties. As interaction progresses, these properties become perceptual references to the function's mathematical structure. At this stage, the artifact becomes transparent, allowing the learner to act through it and reason on the function's properties and structure. This reflects the embodiment of artifact affordances, resulting in a coupling of the body and the artifact in a single entity. The results also emphasize the central role of embodied auditory mediation in supporting mathematical activity. Rather than compensating for the absence of vision, audio–based tools, such as *Audiofunctions*, can support the emergence of new forms of perceptual access grounded in embodied experience, contributing to a more inclusive understanding of mathematical learning.

Further research could explore how this transition unfolds across different mathematical topics and levels of expertise. Examining multiple learners using different forms of sensory mediation, such as auditory and/or tactile, may help materialize how perception–action loops reorganize when visual access is limited or absent. Such studies could deepen our understanding of how embodied coordination with digital tools supports mathematical activity under diverse perceptual conditions.

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