



ICMI STUDY 25

Conference Proceedings

INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION
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Teachers of Mathematics Working and Learning in Collaborative Groups



FEBRUARY 3-7, 2020

LISBON, PORTUGAL

The Twenty-Fifth ICMI Study

Teachers of Mathematics Working and Learning in Collaborative Groups

LISBON, PORTUGAL
UNIVERSITY OF LISBON



FEBRUARY 3-7, 2020

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NETWORKING OF THEORIES FOR A MULTIFACETED UNDERSTANDING ON LESSON STUDY IN THE ITALIAN CONTEXT

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This paper explores the possibilities offered by combining three theoretical frameworks to observe some experiments in different contexts, in the field of mathematics teachers' professional development, based on the Lesson Study methodology and framed into Cultural Transposition, through the Networking of Theories lens. The three researches share a broader goal: studying what might happen when a "foreign object", such as Lesson Study, is introduced into teachers' practices. The specific goal is to explore how dissimilar theoretical frameworks can highlight different aspects related to the cultural transposition of Lesson Study into the Italian context, and how this cultural transposition can improve, modify or strengthen teachers' practices. Does the confrontation with a foreign culture shed light on which teachers' practices and beliefs are more stable, and which are more malleable and subject to change? Findings provide a positive answer to the applicability of LS in the Italian cultural and didactic context.

Lesson Study, in the last years, has been gaining increased attention in the teachers' professional development research field (Bartolini Bussi & Ramploud, 2018), although in Italy it is not very widespread. This research is part of three research projects in Mathematics Education, two at the Department of Mathematics of the University of Turin and one at the University of Salerno. These studies are rooted in a consolidated Italian culture for a meaningful, long-life teachers' professional development, attentive to the cultural and institutional context. The Cultural Transposition framework (Mellone et al., 2019) stresses the need for a careful approach to the confrontation between practices situated in different cultural contexts. This encounter can fuel a reflection on the reasons behind teaching practices, thus fostering the growth of teachers' professionalism. In this paper we take on this challenge, reflecting on three experiments each with its own peculiarities: they have been conducted with prospective and practicing teachers (both in primary and secondary schools) and they assume three theoretical frameworks (Semiosphere and Semiotic of Cultures for primary school teachers, Semiotic Mediation for high school mathematics teachers, Boundary Objects for prospective mathematics teachers). We will show how the Networking of these theories enrich the discussion on the seemingly common findings.

What is Lesson Study?

Lesson Study (LS) is a collaborative methodology for teachers' professional development rooted in the Confucian Heritage Culture. LS is a three-steps cycle: establishment of long-term learning goals and lesson planning, implementation and observation of a research lesson, discussion on the lesson. These steps can be repeated, like a life cycle in which each lesson is the foundation for new growth. In a LS, a group of at least three practicing teachers and in case some university experts and prospective teachers, collaborate to the detailed planning of a one-hour lesson, which will be taught by one of the practicing teachers in his/her classrooms observed by the others, and discussed by the group. On the one side, LS is a culturally situated methodology and it may not be invariant by

translation (in the mathematical sense) between cultural contexts. On the other side, Cultural Transposition proposes “the decentralization of the didactic [and, in our case, teachers’ professional development] practice of a specific cultural context through contact with the didactic practices of different cultural contexts” as a way to bring forward in teachers and researchers the implicit assumptions in which practices are rooted, eventually revisiting them through an enriched point of view (Mellone et al., 2019).

Italian institutional context and Lesson Study

In the Italian context, teachers’ professional development is defined as compulsory, permanent and strategic by the Ministry of Education (law 107/2015), and it is recognised as an opportunity for effective professional growth. The widespread feeling of professional isolation on the part of the teaching community, whose work is becoming increasingly complex from the scientific, humanistic and social points of view, is the main cause for the law to highlight the promotion of collaboration between teachers as a key principle, and encourages professional development in collaboration at the level of the individual school, and at territorial, national and international level. The preparation of quality teachers, as a key mediator of student performances, is not exclusive to the Italian context. It has gained increasing attention in recent years at the international level (OECD, 2009, 2012); in Europe, it is the cornerstone of the Europe 2020 development strategy. Italy already has many teams in which the culture for a meaningful, long-life collaboration in teachers’ professional development is deeply rooted: for mathematics, a never-ending tradition started in the 60s with the *Nuclei di Ricerca Didattica* (Arzarello & Bartolini Bussi, 1998), grew with the *Matematica per il Cittadino* project (MIUR, 2001; 2003; 2004) and currently goes on with many local and national projects coordinated by the Ministry of Education (e.g. m@t.abel, Piano nazionale Lauree Scientifiche). LS can be a further support in the struggle to respond to the demands of the institutions, and another support to the research community in the ongoing development of a culture for collaboration between mathematics teachers.

The three experiments

In the following, we will describe the three experiments designed to understand how to promote, design, and assess relevant collaborative professional development practices for mathematics teachers, each with teachers of different school levels and with different theoretical frameworks.

The first LS experiment is set in a primary school in Piossasco, near Turin. The working group consists in a retired teacher-researcher, four teachers of four different classes, and a researcher. Three complete LS cycles are carried out by three teachers in their 1st-grade classes. The theme of the lesson is the introduction of the ‘plus’ sign and its institutionalization. The goal on children is to understand the concept of addition as the sum of two quantities in its epistemological meaning of putting things together, and relate it to the sign of mathematical language. Later on, another cycle is carried out by the fourth teacher in the 3rd-grade class, in which the activity designed for this LS is part of the educational path that includes the knowledge of weight measurements and the study of state transitions, through experiments with water. The aim is to accompany students in reinvesting their mathematical knowledge and argumentation skills to the transversality of the disciplines. Each teacher implemented the lesson in his or her class. The experiment is observed through a semiotic lens, adapted from the Semiosphere (Lotman, 1990), looking at a space in which we can observe the

dynamics that develop among teachers in the design, implementation of the lesson and in the *a posteriori* discussion. A semiotic lens is used to look for "a truth already present [in the teachers, and] that [only] waited to be recognized" (Sedda, 2006). According to Greimas (Ricoeur & Greimas, 2000), the semiotic point of view provides the scientific knowledge that enables us to investigate that spectrum of knowledge "always known" (and in contrast to the spectrum of "I never thought about it"), but not explicit. In particular, through the introduction of a "foreign object" (the LS methodology) in the usual practices and beliefs of Italian teachers, a process of *deconstruction* is carried out (Bosch & Gascon, 2006, p. 53; Mellone et al., 2019) which "exudes" from the noosphere (Chevallard, 2002, p. 9), influencing the levels of didactic co-determination (Chevallard, 2002, p. 10) and the reflection by teachers. The Semiosphere is in itself a research lens based on collaboration, interaction, in fact it is alleged that no semiotic system can culturally function in isolation.

The second experiment describes a LS trial conducted in Salerno, which involves four teachers from a scientific-oriented high school in Avellino and three researchers from the Department of Mathematics of the University of Salerno. The trial is connected to the well-established tradition of designing Learning Units and carrying out activities within the project Liceo Matematico (Capone et al., 2017): groups of teachers are systematically in contact through meetings with university researchers, to implement collective planning of *ex ante* educational activities and *ex post* analysis of processes. For this LS experiment, the theme "tessellations" is chosen for the learning unit "the art of geometry", connecting with natural sciences and art. Semiotic Mediation (Bartolini Bussi & Mariotti, 2008) is the framework that characterizes the experiment. Two artefacts are used. The first one, used in the Engage phase in Inquiry mode, uses as traces: a sentence, two images and a technological tool (smartphone). The situated texts produced by the students are transcribed on appropriately-made observation sheets. In the final part of the sheets, students are asked to formalize their observations in a mathematical text: each group will therefore provide its own "definition of tessellation". The second artefact is used to solve the real problem: in this case, the traces are cardboard polygons, while no technological tools are used. Once again, the students will write their texts on an observation sheet. The mathematical knowledge is expressed through oral communication. All the implemented activities are socio-semiotic, both because they arise from the sharing between teachers and because they are designed taking into account the Vygotsky perspective of knowledge as a shared experience.

The third experiment involves 29 prospective teachers at the Department of Mathematics of the University of Turin. The aim is to find out the reproducible components useful to implement LS with practicing teachers in the Italian context. The prospective teachers have no previous teaching experience. They worked in small groups: each group is required to create a different activity on continued fractions, from which to draw up a Lesson Plan for a 20-minutes lesson. The lesson is to be performed in front of the researchers and the other prospective teachers, and subsequently discussed within the group. We can stress some differences with usual LS contexts: first, LS is usually performed inside schools and participants have some teaching experience; second, lessons usually last one curricular hour; last, LS is a non-evaluative methodology. As LS is a new methodology for both prospective teachers and researchers, the Boundary Object and Boundary Crossing framework is used to analyse how the two communities act to cope with the novelty, and how LS (the Boundary Object) evolves as a consequence (Star, 2010; Akkerman & Bakker, 2011).

Theoretical Framework

We use part of the Networking of Theories framework (Prediger et al., 2008) as an appropriate (meta)language that makes possible the reference to new conceptual entities connecting our frameworks. In particular, we refer to Radford (2008). He describes a theory as a way of producing understanding and ways of acting based on: *a system P* of basic principles (not a set, for which there is a strong relationship between many of its elements), which includes implicit views and explicit statements outlining the frontier of what will be the universe of the discourse and the research perspective adopted; *a methodology M*, which includes techniques for data collection and data interpretation supported by P, that is, both a theoretical characterization and the very functioning of the methodology (Bernstein, 2000) [the minimum requirements for M are operability and consistency with respect to P.]; *a set Q* of paradigmatic research questions. Table 1 describes the Principles, Methodologies and Research Questions for our frameworks:

Table 1: the three theoretical frameworks

	Semiosphere	Semiotic Mediation	Boundary Objects
Principles	Semiosphere: a multi-cultural dynamic space, interwoven with flows of text, processes of elaboration and understanding of meanings generated by individuals as they interact and know each other. Outside of it there can be neither communication nor language. It is the result and the condition for the development of culture (Lotman, 1990). Its characteristic elements are: heterogeneity and dynamism (it is linked to natural or human aspects of social relations, homogeneity is an anomalous instance); boundary as one of the main mechanisms of semiotic individuation, a porous membrane that marks the passage between "me and the other"; translation as the	At the centre of semiotic mediation is an artefact that embeds mathematical meanings, but is not transparent to embedded meanings. Students, interacting with the mediator, will leave "traces" of their activities (through situated signs). These traces constitute the Semiotic Bundle (Arzarello et al., 2009), that is the dynamic system of signs of various nature (e.g. gestures and words) and of their relationships (e.g. the contemporaneity of a gesture and a word) produced by one or more subjects who interact during the execution of a task. Learning as a teacher mediated social activity. Roots in the vygotskian cultural approach favouring social knowledge.	Boundary as a sociocultural difference leading to discontinuity in action or interaction between communities. Continuity and discontinuity, in the sense that within discontinuity two or more sites are relevant to one another in a particular way. When different communities share a goal, they negotiate a platform at the boundary that allows permeation of practices and preservation of the identity of each community. This crossing might generate tension, which might be the harbinger of new knowledge (Akkerman & Bakker, 2011). Boundary Objects are dynamic objects residing at the boundary, ill-structured with the potential of creating a bridge

	primary mechanism of dialogue and knowledge due to the generating asymmetry, characteristic of the space of the Semiosphere.		between the different communities, robust enough to maintain their identity when they become tailored to local use (Star, 2010).
Methodology	Comparison and analysis of texts, considering all the productions, both of the teachers and of the students, as texts. Context analysis. A posteriori analysis of texts, both verbal and written, produced collectively or individually (transcriptions of audio and video, protocols).	Context analysis. A priori analysis of the semiotic potential of the artefact. A posteriori analysis of texts, both verbal and written, produced collectively or individually (transcriptions of audio and video, protocols).	Introduction of the Boundary Object in a community. Analysis of group dynamics and documentation to investigate how they evolve in the interaction with the Boundary Object. Analysis of the evolution of the Boundary Object when communities act on it.
Research Questions	<ol style="list-style-type: none"> 1. How does Cultural Transposition interact with teachers' beliefs and educational practices? 2. Which specific methodological elements, encountered in the experiments in the light of Cultural Transposition, are highlighted by the different theories and their Networking? 3. Which methodological components of LS are relevant to question 2, with respect to the evolution analysed in question 1? 		

A networking of different theories can be seen as a set of connections involving at least two theories. A connection depends on at least two parameters: the structure of the theories involved in the connection; the purpose of the connection. In the framework of Prediger, Bikner and Arzarello (2008) the panorama of networking, seen as a dialogue between theory and cultures in multi-theoretical research (Bikner-Ahsbahs & Vohns, 2019), is painted by strategies. Since our intention is not to unify theories, but rather to make them communicate with each other, we focus on the goals of the networking strategies: in 'comparing', it is to discover similarities and differences; in 'contrasting' to highlight differences; in 'coordinating', elements from different theories are chosen and put together to investigate a given research problem. In 'combining', the elements chosen do not necessarily show the coherence observed in coordinating theories (it is rather a 'juxtaposition' of theories - Radford, 2008). In our three researches, with different theoretical frameworks and therefore with principles and methodologies that are not entirely congruent, we answer the same research questions. For this reason, to achieve our goal we will use the strategy of combining theories.

Findings

Because of space constraints, data supporting these findings will be presented in a future paper. Each theory has its own specificity. Combining three different points of view on similar dynamics,

therefore, has the same effect that panting a complex object from three different positions of view can have: it highlights aspects and details, relations between components, dynamics that do not belong to the visual cone of a single observer. With our theoretical frameworks, we focus our attention on a common space: the way in which Cultural Transposition allows us to contextualize and deconstruct (Bosh & Gascon, 2006) teachers' beliefs and practices. The three frameworks conceive, in different ways, LS as an element that interacts with the Italian context and its components.

As a *Boundary Object*, LS is a dynamic object, moving on the boundary between communities of practice. The analysis focuses on the meeting at the boundary between the community of prospective teachers and the one of researchers, which also metaphorically represents the practicing teachers. On the one hand, the two communities collaborated on the reflection on LS, which allowed the researchers to highlight some components of the methodological object that had remained implicit in the brokering of LS from the Japanese cultural community to the Italian one. On the other hand, the researchers were able to observe how LS helped the *boundary crossing* of the prospective teachers towards the practicing teacher's community, making sense of a different perspective. Moreover, it was possible to observe how the encounter with the robust components of LS (Star, 2010) developed a reflection of the prospective teachers on their own meta-didactic praxeologies (Arzarello et al., 2014), possibly transformed into a hybrid between the long-term approach, typical of the Italian context, and the fine analysis used in LS. The analysis of the evolution of both communities of practice and of the Boundary Object itself, allowed the researchers to highlight the potential of LS as a Boundary Object to trigger some dialogic mechanisms for professional growth indicated by Akkerman & Bakker (2011). The community of researchers developed new praxeologies for the introduction of LS in the Italian institutional context.

Immersing ourselves in the visual cone of the *Semiosphere*, we see how the LS methodology, becoming part of the mathematics class' semiosphere, allows the deconstruction of practices and beliefs, so producing a new awareness. In particular, it allowed to look at the collaboration between teachers, and thus at the elaboration, exchange, and archiving of mathematical knowledge and professional development, as mutually inclusive continuous texts. The texts are written (e.g. Lesson Plan), graphic (e.g. drawings of students or graphic representations of teachers), oral (e.g. dialogues in the various phases of the LS), technologically mediated (e.g. worksheets, machines), embodied (e.g. interaction in the classroom), institutional (e.g. curricula), local (e.g. specific epistemological, didactic and pedagogical needs), and others. Through their heterogeneity in mutual continuity, the semiospherical texts allow to keep connected aspects that would seem distant. Distance (understood here in a cultural and sfemiospheric sense - Lotman, 1990) would generate a loss of meaning.

Semiotic Mediation allows us to reflect on the importance of the teacher's role in the appropriate choice of artefact linked to its semiotic potential, and on the importance of the teacher's role in the management of discussion and sharing of individual signs; the teacher also seems more aware that better time management involves better class management. In the teaching practice, LS, shared with the whole teaching community, seems to contribute not only to the professional growth of the experimenter teacher, but transfers to the whole community the refined skills and the acquired awareness of their role in guiding the discussions in the classroom. The identification shared by the experimenter teachers with the semiotic potential of the proposed artefact was the necessary background to its use in the classroom. The careful planning of the didactic intervention, of the

possible tasks and the didactic organization foreseen into LS allowed to assume the right semiotic perspective to focus on the production of signs and on the process of transformation of these signs. The teacher, after just one LS cycle, becomes more aware of the choice and use of the artefact to make it functional to semiotic mediation.

The combination of the three theoretical lenses allows us to go beyond the single point of view. If the frameworks of Semiosphere and Boundary Object carry out a meta-analysis of LS in relation to the context in which it operates, identifying the points and tools for intervention in a complementary way, Semiotic Mediation provides us with the tools to observe LS in its operational practice, reading and interpreting the didactic action of the teacher, and therefore promoting an analysis of the effectiveness of LS itself as a professional development practice. In this sense, in absolute consonance with the Semiotic Mediation, the lens of the Semiosphere allows us to focus on the patterns of reasoning that the students use, and on the essential components of socialization of reasoning in building meaning. Starting from the three experiments and through the juxtaposition of the three lenses, we were able to highlight some of the specific teaching practices within the Italian context. From an institutional point of view, our teachers have expressed great difficulties in organizing the time in which to do their work and, at the same time, a need for flexibility with respect to the management of time in the classroom. Moreover, in particular due to the cultural and institutional context but also to emotional aspects, teachers feel the need to adequately respond individual students' needs, something possible only by giving the right importance to design, planning, and assessment of teachers' actions. Finally, from a content point of view, we found deeply rooted fears about the management of mathematical misconceptions, alongside the underestimation of the possibilities offered, in this sense, by research in Mathematics Education.

The LS methodology has contributed to this study, thanks to the new perspective with respect to teachers' meta-didactical and collaborative praxeologies (Arzarello et al., 2014), providing them a tool for microanalysis of the phases of the lesson in a context accustomed, for historical and institutional reasons, to the design and analysis of long-term development strategies. The encounter with other people's practices within LS is an opportunity to observe and reflect on one's own different praxeologies. The apparent contrast between a fine lesson planning and the attention to the needs of the individual student accentuated the careful design of the didactic intervention. The detailed and collaborative design of possible tasks and the didactic organization provided by LS have allowed to take the right semiotic perspective to focus on the production of signs and on the process of transformation of these signs.

Conclusion

In this paper, we have used part of the Networking of Theories framework (Prediger et al., 2008) as a (meta-)language that made possible the connection and harmonization between our three theoretical frameworks (Semiosphere, Semiotic Mediation and Boundary Objects). LS methodology in the Italian context allowed us to closely study the practices of prospective and practicing teachers in collaborative contexts. By combining the three theoretical lenses we have tried to extend the experimentation of LS to different contexts (practicing teachers in primary and secondary school and prospective teachers at university level). The results highlight the collaborative dimension in teaching/learning practices as a possible key for a real reform of teaching, seeking and creating connections between teaching practices of different school segments; the collaboration between

school and academia can be an added value towards more conscious teaching practices in the light of the research results. This was possible thanks to a careful cultural transposition of LS into our educational context, which provided a solid methodology for teachers' collaboration within the institutions: the data of the three different experiments on different school levels from different points of view provided a novel understanding on how to promote, design, and assess relevant professional development practices for mathematics teachers. The combined findings seem to have provided further support to the applicability of LS in our didactic praxologies. Overall, LS seems to be replicable as an effective teachers' professional development practice, suggesting the potential of a not only horizontal collaboration. A training that flourishes from below can be of support to the communities of practicing teachers and a stimulus for prospective teachers. Observing one's own work through the practices of others allows a more conscious reflection on one's own practices, laying the foundations of a modern teachers' professional development.

Acknowledgements The authors are deeply thankful to professors Ornella Robutti and Ferdinando Arzarello, University of Turin, for their guidance and help in drafting this paper.

References

- Akkerman, S. F. & Bakker, A. (2011). Boundary Crossing and Boundary Objects. *Review of Educational Research*, 81(2).
- Arzarello, F. & Bartolini Bussi, M. G. (1998). Italian Trends in Research in Mathematical Education: A National Case Study from an International Perspective. In A. Sierpiska & J. (eds.). *Mathematics education as a research domain: A search for identity*. (pp. 243-262) Springer: Dordrecht.
- Arzarello, F., Paola, D. Robutti, O. & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, 70(2), 97-109.
- Arzarello, F., Robutti, O., Sabena, C., Cusi, A., Garuti, R., Malara, N., & Martignone, F. (2014). Meta-didactical transposition: A theoretical model for teacher education programmes. In *The mathematics teacher in the digital era* (pp. 347-372). Springer: Dordrecht.
- Bartolini Bussi, M. & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. D. English (Ed.) *Handbook of international research in mathematics education* (pp. 746-783). New York: Routledge.
- Bartolini Bussi, M. G. & Ramploud, A. (2018). *Il lesson study per la formazione degli insegnanti*. Roma: Carocci.
- Bernstein, B. (2000). *Pedagogy, symbolic control, and identity: Theory, research, critique*. Lanham: Rowman & Littlefield.
- Bikner-Ahsbals A., Vohns A. (2019). Theories of and in Mathematics Education. In H. Jahnke & L. Hefendehl-Hebeker (Eds.) *Traditions in German-Speaking Mathematics Education Research* (pp. 171-200). Cham: Springer.
- Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI bulletin*, 58, 51-65.
- Capone, R., Rogora, E., & Tortoriello, F. S. (2017). La matematica come collante culturale nell'insegnamento. *Matematica, Cultura e Società. Rivista dell'Unione Matematica Italiana*, 2(3), 293-304.
- Chevallard, Y. (2002). Organiser l'étude. 3. Écologie & regulation. In J.-L. Dorier et al. (Eds.) *Actes de la 11e École d'Été de didactique des mathématiques* (pp. 41-56). Grenoble: La Pensée Sauvage.
- Lotman, Y. M. (1990). *Universe of the Mind. A semiotic theory of culture*. London: IB Taurus.
- Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM*, 51(1), 199-212.
- MIUR (2001; 2003; 2004). *Matematica 2001-2003-2004. Attività didattiche e prove di verifica per un nuovo curricolo di matematica*. Scuola Primaria e Scuola Secondaria di I grado; Ciclo secondario; Quinta classe. Lucca.
- OECD (2009; 2012). *OECD Annual Report 2009; 2012*, OECD Publishing, Paris.
- Prediger, S., Bikner-Ahsbals, A. & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: First steps towards a conceptual framework. *ZDM*, 40(2), 165-178.
- Radford, L. (2008). Connecting theories in mathematics education: Challenges and possibilities. *ZDM*, 40(2), 317-327.
- Ricoeur, P. & Greimas, A. J. (2000). *Tra semiotica ed ermeneutica*. (F. Marsciani, Trans.) Roma: Meltemi.
- Sedda, F. (2006). Imperfette traduzioni. In Lotman, J. M. *Tesi per una semiotica delle culture* (pp. 7-68). Roma: Meltemi.
- Star, S. (2010). This is not a boundary object: Reflections on the origin of a concept. *Science, Technology, & Human Values*, 35(5), 601-617.