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ABOUT THESE PROCEEDINGS

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EXPLORING MATHEMATICAL FUNCTION GRAPHS THROUGH SONIFICATION: A HANDS-ON WORKSHOP WITH AUDIOFUNCTIONS

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This workshop offered participants a hands-on exploration of mathematical function sonification using Audiofunctions, a multimodal web application that enables exploration of function graphs through visual and auditory feedback. Participants actively engaged with both continuous and discrete sonification techniques, experiencing how different sound features (e.g., pitch, panning, volume, earcons) can represent mathematical relationships. Through three distinct interaction modes and various timbres, participants discovered how auditory feedback can enhance mathematical understanding and create more inclusive learning environments.

Keywords: Function graphs, sonification, visual impairments, inclusion, SONAIRGRAPH.

WORKSHOP OVERVIEW

Mathematical visualization has long been a cornerstone of mathematics education, but what happens when we engage our auditory senses alongside visual representation? This interactive workshop introduced participants to *Audiofunctions* (<https://audiofunctions-plus.netlify.app/>), a multimodal web application originally developed by the Laboratory "S. Polin" at the University of Turin and currently being researched within the SONAIRGRAPH Erasmus+ Project¹. *Audiofunctions* represents a significant advancement in accessible mathematics education, originally designed to support students with visual impairments but proving valuable for all learners. This software transforms mathematical function graphs into an experience of content signification through *sonification* techniques (Hermann et al., 2011). We believe that it could represent a tool to support development of *covariational reasoning* (Thompson & Carlson, 2017), since "its sonification offers a dynamic but temporally sequential mode of access to function graphs" (Manolino et al., in press).

WORKSHOP ACTIVITIES

Phase 1: Getting started with Audiofunctions (30 minutes)

During the initial activities, participants freely engaged with the software, thereby uncovering which aspects of sonification could be readily perceived through preliminary exploration. Importantly, no explicit instruction regarding the application's specific functionalities was provided. This choice was deliberately adopted to investigate how novice users interpret the interface based solely on their direct interactions. Guiding questions such as: "Which auditory elements can you identify?"; "What mathematical relationships do you believe they represent?"; "Which function properties emerge intuitively?" facilitated the session.

Phase 2: Discovering function properties through comparison (30 minutes)

This time, several pairs of functions were explored. For each pair, participants alternated between the two functions using the 'switch functions' button, listening to one sonification then the other to enable comparison. The aim was to understand whether the sound representations of two functions allow the

recognition of similarities and/or differences that, in a subsequent phase, can be formalized and named with specific mathematical terms.

Phase 3: Educational implications and final thoughts (30 minutes)

The final phase engaged participants in discussing the results of the hands-on exploration, as well as practical classroom applications. Drawing on recent research in mathematics learning mediated by digital artifacts (Baccaglini-Frank et al., 2025), the theoretical foundations that underpin Audiofunctions and its application in the mathematics education field were discussed.

PARTICIPANTS' REFLECTIONS AND FINDINGS

Participants demonstrated varied levels of awareness regarding the sonification design. All respondents identified the fundamental pitch-to-function-value mapping, with one noting: "the pitch is connected to the function values (the higher the point, the higher the pitch)". The stereo panning feature was also recognized, observing that "x-values are connected to the left-to-right stereo sound". They also identified several earcons and the background noise (described as "hissing sound") was noted to appear "below x-axis only", while volume variations indicated "further from line = quieter volume". The variety of interaction methods appeared to influence what features participants discovered, with one noting he used all for a comprehensive exploration.

The comparison between continuous (clarinet) and discrete (guitar) sonification modes emerged as particularly significant. One participant noted: "The guitar give[s] me the feeling [of] the clear perception of values rather than their changing, such as constant, decreasing, increasing, and concave up... While I only perceived the changing of function through the voice of clarinet". A second one observed: "Using discrete mode helps to interpret the shape of the graph. You can hear different intervals between notes to judge the gradient". Moreover, when comparing $|x|$ and $0.5x^2$, another participant suggested that "speed changes to indicate rate of change of the curve. So a faster speed demonstrates a faster change of gradient".

These reflections suggest that participants developed multiple strategies for extracting information from auditory representations. The variety of features identified indicates that multimodal sonification successfully provides redundant encoding of mathematical relationships. The preference for discrete mode when analysing function properties suggests its particular value, while continuous mode may be more suited to perceiving global function behavior and overall trends.

ENDNOTE: ¹Project titled "Sonairgraph - SONification for Accessible and Inclusive Representation of GRAPHS in Education" (<https://sonairgraph.unito.it/>), within the ERASMUS + programme, in response to Call 2024 Round 1 KA2 - KA220-HED - Cooperation partnerships in higher education, CUP B63C24000960006.

REFERENCES

- Baccaglini-Frank, A., Carotenuto, G., Funghi, S., Lisarelli, G., & Miragliotta, E. (2025). Digital artifacts in mathematics education: how can we study the learning processes they promote?. *Bollettino dell'Unione Matematica Italiana*, 18(1), 17-48. <https://doi.org/10.1007/s40574-024-00439-2>
- Hermann, T., Hunt, A., & Neuhoff, J. G. (Eds.) (2011). *The Sonification Handbook*. Logos Verlag Berlin.
- Manolino, C., Funghi, S., Ducci, M., Brunetto, E., & Bernareggi, C. (in press). Navigating function graphs with audio: an investigation into the narratives of a blind expert using Audiofunctions. In *Proceedings of the 14th Congress of the European Society for Research in Mathematics Education*.
- Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In *Compendium for research in mathematics education* (pp. 421-456).