

# Navigating function graphs with audio: an investigation into the narratives of a blind expert using Audiofunctions

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*This study examines Audiofunctions (AF), a multimodal interactive digital artifact designed to represent graphs of real-valued functions of a real variable. AF displays function graphs and uses sonification to represent the coordinates of a moving point as the independent variable changes. The paper discusses key theoretical considerations underlying the design of AF, related to a covariational perspective on functions and function graphs. Using a commognitive perspective, it analyses the case of an adult blind user, an expert in mathematics, in interaction with AF. The study investigates the narratives she produces, the function realizations involved other than the auditory one, and how they are related to the function characteristics, providing insights into the potential impact of AF.*

*Keywords: Function graphs, multimodality, inclusion, covariation, visual impairments.*

## Introduction

The notion of inclusive education, aimed at providing high-quality education for all students, regardless of disabilities, gained attention in last decades (e.g., Ainscow, 1997). Nevertheless, ensuring the right to education for all faces persistent challenges, such as improving accessibility of materials and content. Despite advancement of assistive technologies, while accessible formats such as haptic or audio options are available for digital textual content (e.g., braille display – a device translating text written on screen in Braille characters accessible to user by touch – or screen readers), there is still research going on to find a general solution for accessible graphs and formulas (e.g., Shahira & Lijiya, 2021). Moreover, a lack of educational research on use of these tools remains, even if some studies showed that using sonification for teaching-learning spatial concepts seems promising (e.g., Damsma, 2024). This paper focuses on the issue of ensuring sustainable accessibility for function, a fundamental concept in scientific education. In functions teaching-learning, interpreting information embodied in function graphs or, conversely, using available information about an assigned function to graph it, is a crucial competence (e.g., Baccaglini-Frank et al., 2023). Access to function graphs' information is essential for understanding functional relationship between dependent and independent variables, but function graphs pose accessibility challenges due to their graphical nature. In this context, Laboratory “S. Polin” of the University of Turin is developing since 2014, and now with the Erasmus+ project “Sonairgraphs”, a software solution called *Audiofunctions* (AF), publicly available since 2019 as a web-based application designed to generate accessible single-variable function graphs through use of sound (<https://audiofunctions.netlify.app/>). In these graphs, at visual level, the cartesian function graph is displayed, with a bullet point moving on it depending on the position of its coordinates; at auditory level, a sound is produced whose characteristics depend on the position of the bullet point on screen (e.g., move mouse pointer wearing headphones to hear  $f(x)=\sin x$ , linked at: <https://linkshort.acesee.it/sinx>). Therefore, sonification in AF allows visually impaired students to access functions by “discovering” gradually their behaviour through sound.

Next to accessibility issue, literature underscores learning difficulties encountered by students, with or without impairments, in making sense of function graphs. Antonini et al. (2019) emphasise that in literature students show fewer difficulties when they deal with “numerical evaluation or prescribed algebraic manipulation, but they tend to encounter difficulties in analysing or even describing a function’s behaviour by looking at its Cartesian graph” (p. 2). Function graphs are often perceived by students as static representations “in which the variations, and the relation between variations – *covariation* – related to functional dependency remain hidden” (Antonini et al., 2019, p. 2, our emphasis). In this paper, we address the ability to envision the relationship between simultaneous variation of values of independent and dependent variables. Prior research (e.g., Thompson & Carlson, 2017) highlighted that covariational perspective on quantities dependence was characteristic of first conceptualisations of function in mathematics’ history, even if it was subsequently abandoned. However, Thompson and Carlson (2017) argue that “continuous variation and covariation seem to be epistemologically necessary for students and teachers to develop robust conceptions of functions” (p. 35). In other words, familiarising students with function graphs alone does not guarantee covariational reasoning skills’ development, essential for understanding function concept. While ensuring access to this content for all students is critical, it may not be sufficient to foster meaningful learning. It could be worth studying whether AF can be exploited not only as a tool for ensuring accessibility, but as a tool to support development of covariational reasoning for *all students*, since its sonification offers a dynamic but temporally sequential mode of access to function graphs (see Audiofunctions’ design paragraph). For sighted students, this dynamic representation contrasts with the static, comprehensive, visual graph. Sound dynamism is replicated at visual level through bullet point’s movement, allowing to see the visual graph as a dynamic time-related evolving trajectory. This would make AF suitable for designing inclusive learning activities.

## Theoretical Framework

We take a *commognitive* perspective (Sfard, 2008) consistent with previous studies on interaction with digital artefacts investigating development of meaningful narratives in the context of functions and variables (e.g., Baccaglioni-Frank, 2021; Antonini et al., 2019). In this perspective, the term "discourse" is intended as a form of communication characterized by specific features, among which we find specialised vocabulary (e.g., terms used to name mathematical objects, as “function”) and unique *visual mediators* (such as numerals, symbols, or graphs). Constructing mathematical objects involves a *saming* process, i.e. naming seemingly unrelated things that are seen as equivalent in certain contexts (e.g., "linear function" can refer to “ $3x$ ”, a straight line, or a table of paired couples of numbers). These things become different *realizations* of the same mathematical object. A mathematical object is thus to be understood as a purely discursive object that, in a given discourse, is constituted by the set of its realisations. We emphasize here that realizations of mathematical objects are defined by Sfard without specifying their perceptual nature (Maffia et al., 2023), hence potentially including also auditory mediators. “[Mathematical] discourses are made distinct [...] by the form and outcomes of their processes, that is, the *routines* and *endorsed narratives* that they produce” (Sfard, 2008, p. 161). In mathematical discourse, experts “interpret most task-situations as requiring a (re)formulation and endorsement of a particular type of mathematical narrative” (Sfard, 2020, p. 96). “*Narrative* is any sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects, that is subject to *endorsement* or rejection with

the help of discourse-specific substantiation procedures” (Sfard, 2008, p. 134). Baccaglini-Frank et al. (2023) discuss how experts talk about function behaviour starting from its graph, emphasizing their exploitation of a covariational perspective that enables them to make sense of the function beyond what they “just” perceive with sight. Different realizations of the same signifier in a discourse are related precisely thanks to discourse rules allowing the translation of endorsed narratives about a realisation into ones about the other. AF offers multiple realizations of the same function, involving visual mediators (algebraic expression, Cartesian graph) and an auditory mediator (sonification). The latter, covariational and dynamic, provides a unique representation for visually impaired users, who can integrate it with other, more common but usually non-dynamic, representations (e.g., tactile ones). In this paper, we want to focus on how a blind expert recognise AF’ sonification as a realization of function. Then, we address the following research questions: What other realizations of the function are present in the expert’s discourse dealing with AF? How does she translate her endorsed narrative on function into new narrative on sonification?

### Audiofunctions’ design

The first version of AF, *Audiofunctions.web*, was thoroughly investigated in technical aspects, usability, and accessibility (see Ahmetovic et al., 2019). Its sonification technique maps function related information to sound features: if we name  $P=(x, f(x))$  the moving point, sonification conveys the current position of  $x$  mapping the range on  $x$ -axis from left to right with auditory *panning* (i.e. an  $x$  visually closer to the left graph bound is heard more by the left ear), and the value of  $f(x)$  mapping the range on  $y$ -axis with sound *pitch* (the perceived sound frequency). Pitch is central to sonification, the human auditory system is sensitive to its variations (Dubus & Bresin, 2013). In this way, exploring function from left to right, users can “hear” the displayed portion of graph as the covariation of sound panning and pitch. Originally emerging as a tablet application, the prototype of *Audiofunctions.web* (Bernareggi, et al., 2016) underwent evaluation by comparing the interaction experience of blind users with respect to their accustomed use of tactile drawings.

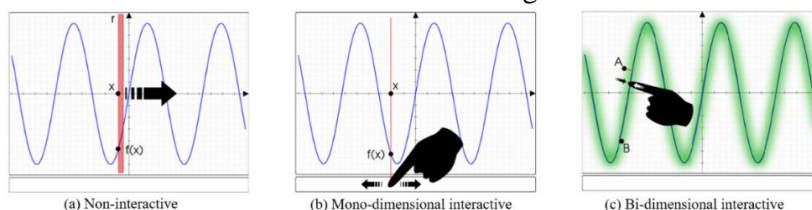


Figure 1. Function exploration modes (Bernareggi et al., 2016, p. 8)

Users freely engaged with three available interaction modes (see Figure 1a,b,c respectively): *non-interactive mode*, where pressing “b” key the software sonifies automatically the entire function, varying  $x$  from left graph bound to right one; *mono-dimensional interactive mode*, where, in a dynamic exploration, sonification is reproduced as users move with keyboard arrows or slide the pointer along a horizontal line; *bi-dimensional interactive mode*, where spatial relationships can be grasped by difference in volume since that the highest volume occurs on function line. Bernareggi et al. (2016) indicate *Audiofunctions.web* as more efficient in terms of response time compared to tactile methods. This highlights the software as a valuable tool for enhancing information accessibility. AF is the latest version developed to specifically facilitate its educational implementation. Among other enhancements, AF implements two distinct types of sonification: *continuous sonification* with clarinet and sine wave timbres, and *discrete sonification* with guitar timbre. There are two key

differences between them: *pitch density* and *threshold passing*. Pitch density refers to the fact that in discrete sonification a predetermined number of notes is established *a priori*, then the displayed  $y$ -axis range is partitioned into a corresponding number of subintervals of equal width and each interval is then associated with a guitar musical note. In continuous sonification, instead, the notes are distributed continuously along the displayed  $y$ -axis range. The threshold passing is a sonification strategy that leverages the lower pitch density in the discrete sonification: instead of sonifying a value on the  $y$ -axis (as in continuous sonification), it sonifies the threshold passing from one of the subintervals on the  $y$ -axis to another. The  $x$ -axis is also discretised into subintervals  $[x_1, x_2], \dots, [x_{n-1}, x_n]$ , with each position  $x_i$  mapped to the note of the  $y$ -axis subinterval where  $f(x_i)$  falls. However, if  $x_i$  and  $x_j$  are two subsequent positions sonified by AF with guitar,  $j$  is the first index greater than  $i$  where the note associated to the subinterval containing  $f(x_j)$  differs from that of  $f(x_i)$ . Therefore, users hear a pause between two successively produced notes, since all the  $x_t$  comprised between  $x_i$  and  $x_j$  do not give place to any sound by AF. AF' design is grounded in explicit educational intentionalities, consistent with principles of a well-designed digital environment (Baccaglioni-Frank, 2021). This approach, informed by literature on mathematical content sense-making, is crucial for ensuring that technology not only provides accessibility but also supports users in achieving autonomy and fosters cognitive engagement. AF' design in the continuous sonification mode is inspired by a strand of research by Italian researchers, focusing on the use of digital artifacts, called *dynagraphs*, where two ticks moving along one or two straight lines represent the independent and dependent variable (e.g., Baccaglioni-Frank et al., 2023; Antonini & Lisarelli, 2021; Antonini et al., 2019). In many experiments the two ticks have a functional relationship unknown to users, who can act directly only upon the tick representing independent variable, thereby indirectly affecting the movement of the tick representing dependent variable. Baccaglioni-Frank et al. (2023) emphasise this asymmetry in dragging ticks, arguing that it supports students' development of a covariational view of variables' functional dependency, that they later recognise in Cartesian function graphs. Antonini and Lisarelli (2021) discuss a task sequence, where Cartesian function graph is generated as trajectory of a point of coordinates  $(x, f(x))$ , dragging the tick representing  $x$ . Considering AF in mono-dimensional interactive mode (Figure 1b), AF mirrors aforementioned asymmetry between variables, since users can directly act on the independent variable, changing the moving point position, and thus enabling users to interpret function graph as trajectory of that point. AF differs from dynagraphs in the fact that trajectory is multisensorially rendered maintaining visible the static Cartesian representation.

## Methods

To answer our question, we carried out some interviews with adults with visual impairments, experts in scientific disciplines, to collect data about their ways of interacting with AF when exploring different kinds of functions. Since our aim was to portray blind users' point of view, we opted for an exploratory approach and a case study design, similar to previous studies concerning blind solvers (e.g., Maffia et al., 2023; Miragliotta et al., 2023). Despite their familiarity with function concepts, participants were unaccustomed to AF ways to represent functions. In this paper, for reason of space, we examine the case of Miriam (pseudonym), a woman in her thirties who works as a business analyst and graduated with an honour degree in mathematics. She has been blind since birth and does not use speech synthesis but reads Braille. She knew that she was going to experiment a software for representing functions but did not know AF; she was used to their algebraic expression and to their

tactile graph. The interview was conducted through an online remote calling; the interviewer was the first author, who shared through the chat of the videocall platform links related to different functions already setted in AF, provided one-by-one during the conversation. For interacting with her computer, Miriam could use tools as screen reader, keyboard, or braille display. The interview was semi-structured: it involved a first phase where she was introduced to the software, and she was asked to freely explore AF setted with some non-standard functions. This was done to favour a focus on the description of the qualitative characteristics of the function behaviour rather than on an attempt to guess its algebraic formula. In a second phase, then, Miriam was asked to explore: different constant functions (e.g.,  $y=0$  and  $y=3$ ); linear functions with different slopes (e.g.,  $y=x+3$ ,  $y=\frac{1}{2}(x+3)$ ,  $y=-x+3$ ); functions having similar behaviour with respect to some properties (e.g.,  $y=x^4$ ,  $y=x^2$ , that are decreasing for  $x<0$  and increasing for  $x>0$ , have a point of minimum for  $x=0$  and tend to infinity for  $x$  going to  $+\infty$  and  $-\infty$ ); functions whose domain is not the whole set of real numbers (e.g.,  $y=\sqrt{x}$ ). The displayed  $x$ -axis and  $y$ -axis ranges for all the functions were pre-determined and set as follows:  $-16<x<16$ ;  $-9<y<9$ . The choice of functions with similar behaviour was aimed at understanding aspects of sonification used by Miriam to distinguish similar functions from each other. The interview was recorded and transcribed. Accordingly with previous studies exploiting a commognitive perspective (e.g., Baccaglini-Frank, 2021), Miriam's discourse was analysed by looking for words related to different function realizations (e.g., algebraic, visuo-spatial, etc.) and their characteristics, and for words related to transitions between different kinds of realizations.

## Data analysis

Miriam conducted the interview using the braille display to modify sound settings or switch functions, and using keyboard keys to explore function graphs. In Table 1 an excerpt is reported where Miriam explores graphs of the constant functions  $f(x)=0$ .

**Table 1. Miriam considerations on  $f(x)=0$  graph** (available at: <https://linkshort.acesee.it/afy0>; wearing headphones is recommended, and use "i" key to switch from discrete to continuous sonification and viceversa)

Line	What is said	What is done
1		<b>Miriam</b> presses the "b" key, and AF plays a single note with the guitar timbre while the red dot moves from left to right along the entire line [graph of the function $f(x)=0$ ].
2	<b>Miriam:</b> But how? Is it already over?	
3	<b>Interviewer:</b> Yes. I mean, it's not true that it's over, but it has a strange "thing" here; you should..	<b>Miriam</b> , acting on the settings slide bar, changes the timbre from guitar to clarinet.
4	<b>Miriam:</b> ...it's all at the same pitch.	<b>Miriam</b> presses the right arrow key several times. AF plays, for each key press and while the red dot moves step by step, the same note previously reproduced.
[...]		
11	<b>Miriam:</b> It's all a horizontal function.	
12	<b>Interviewer:</b> How did you figure it out?	
13	<b>Miriam:</b> The sound was always the same.	

The first interaction mode used by Miriam here is non-interactive, using the "b" key, with guitar timbre; she appears confused by what she hears ([2]), and decides to switch the timbre to the clarinet (continuous sonification). She needed the change in sonification to recognise that the pitch was constant ([4]), together with a change in the interaction mode, from non-interactive to mono-dimensional. Continuous sonification in a mono-dimensional interaction mode probably made her experience a multimodal interaction to analyse the function graph, involving her panning-related and

proprioceptive strategies using arrow keys to orientate herself on the graph along the  $x$ -axis, and her auditory perception of sound pitch. In fact, it's worth noting that Miriam's discourse passes from a narrative about the constant pitch she hears ([4]) – aspect of the auditory realization of  $f(x)=0$  – to a narrative about function through an adjective pertaining to discourse on *spatial* level (“a horizontal function” in [11]). When asked why, she states again her perception of constant pitch (narrative about sound in [13]). Miriam's implicit association of the constant sound pitch (one dimensional) with the overall two dimensional shape of the cartesian graph of a class of functions means that she is interpreting pitch as dependent variable and left-right direction as independent variable. Moreover, she juxtaposes an adjective describing the cartesian graph at spatial level (“horizontal”) with the signifier “functions”, which indicates identification of the function with its graph, that becomes a spatial realization of the function for her. In Table 2 Miriam interacts with non-constant functions.

**Table 2. Miriam considerations on the graphs of  $f(x)=x+3$ ,  $f(x)=\frac{1}{2}(x+3)$  and  $f(x)=-x+3$**  (available respectively at: <https://linkshort.accesee.it/x3>, <https://linkshort.accesee.it/12x3>, <https://linkshort.accesee.it/3x>)

Line	What is said	What is done
38	<b>Miriam:</b> This one is diagonal! Going upwards. It starts from the bottom and goes up to the top.	The $f(x)=x+3$ graph is displayed. The selected timbre is guitar. <b>Miriam</b> presses the "b" key. AF plays a complete scale of notes while the red dot moves from left to right along the line
39	<b>Interviewer:</b> Ok. Switch the function again so we can hear the next one, and then you can tell me about the differences.	
[...]		When the $f(x)=\frac{1}{2}(x+3)$ graph is displayed, <b>Miriam</b> presses the "b" key. AF plays a complete scale of notes, narrower than the previous one and with longer pauses between the notes, while the red dot moves along the line
44	<b>Miriam:</b> It has a lower slope.	
45	<b>Interviewer:</b> It has a lower slope. And can you tell me anything else?	<b>Miriam</b> plays the function with "b" again. And then she thinks for a while in silence.
46	<b>Miriam:</b> No.	
47	<b>Interviewer:</b> No. How do you perceive the difference in slope?	
48	<b>Miriam:</b> It's a slope that lasts longer. It rises less steeply.	
49	<b>Interviewer:</b> Ok. Switch the function again.	When $f(x)=-x+3$ graph is displayed, <b>Miriam</b> presses "b" key. AF plays the complete scale of notes sonified for function $f(x)=x+3$ but starting from the higher note to the lower one, while the red dot moves from left to right along the line
50	<b>Miriam:</b> It is a descending line. I mean [she seems to correct herself], with a negative slope (“coefficiente angolare” in Italian).	

Despite her return to non-interactive mode with guitar timbre, Miriam keeps speaking about what she hears using words pertaining to discourse on a spatial level (e.g., “diagonal”, “upwards”, “bottom”, “top” in [38]). Miriam uses terms related to vertical dimension to describe sound behaviour (“upwards”, “bottom”, “top” in her narrative in [38]) but the overall function behaviour is summed in an adjective about two-dimensional space (“diagonal”, in [38]). We interpret this as a sign that Miriam is relating pitch with some other variable, maybe time or left-right dimension. Moreover, the dynamic connotation of verbs such as “going upwards”, “starts from... and goes up...”, “rises”, shows that she is associating the dynamic pitch changes with a trajectory on the plane – similarly to endorsed narratives of sighted experts reading static functions graphs (Baccaglini-Frank et al. 2023). In [48] she uses expressions related to time (“lasts longer”) and speed (“rises less steeply”), that perhaps refer

to discrete sonification features: the shorter the pause between two successive sonified notes, and the further apart they are on scale, the steeper the function is. When she switches to  $f(x)=-x+3$ , she introduces the term “slope” ([44]), which pertains again to discourse on a two-dimensional space. In [50] Miriam passes to the formal Italian term for line slope, a shift that we interpret as a way to make her discourse more formal, since she is aware that she is talking to a mathematician. Therefore, we infer that probably here she is implicitly referring, for the first time, to algebraic function realization.

## Discussion and conclusions

The interview with Miriam, expert in mathematics, confirms that AF emerges as a valuable tool, providing a comprehensive qualitative overview of functions’ behaviour. Miriam seems to implicitly refer to functions realizations she was used to when she was a student: her recurring to words related to two-dimensional space seems to be connected to her knowledge of the cartesian function graphs – developed thanks to the use of tactile graphs. Talking about the function slope, she seems to implicitly refer also to the function algebraic expression. She passes mainly from auditory realization of AF to cartesian graph, in analysed extracts, often linking sonification with the overall two-dimensional shape of function graph, and describing both the sound behavior and the cartesian graph in terms of dynamic variations (e.g., in [38]), a fact that shows Miriam’s mastery in covariationally reading the function and interpreting the variation of the sound pitch in dependence on time or on shifts in the left-right dimension. This recalls what is shown by the interviews with experts in Baccaglini-Frank et al. (2023) but in a reverse perspective: sighted experts can read the graph, a static image, as a dynamic trajectory of a moving point. Miriam, otherwise, is able to read a dynamic sound realization of function in AF as a two-dimensional visuo-spatial configuration, “synthesizing” overall sound behaviour. In this perspective, AF seems a potential tool for developing a covariational view of function graphs for students with and without visual impairments, since it allows to work on the connections between a dynamic, covariational function realization and a static synthetic realization. It should be clarified whether and how AF should be used combined with other function realizations, such as tactile graphs, for coordinating auditory mediators with visuo-spatial ones. There is more to understand about how people with visual impairments perceive haptic/auditory information, but Miriam’s case suggests that, through AF, users can qualitatively grasp some function features, such as points on the graph where the function is higher compared to other points. Users can compare functions in terms of their growth or decline rates, leveraging the concept of speed of discrete sonification. Combined interaction of discrete and continuous sonification can offer users an initial embodied conceptualisation of derivative, through an auditory completion process when listening to two successive notes. Users can also grasp functions monotony, even if it may not be always easy to distinguish between functions with similar behaviours (about  $y=x^4$ , Miriam commented “It goes very fast [...] at the beginning is at infinity then goes down for a moment and goes up and back to infinity [...] But how do I know if it’s a V, or if it’s like a basin?”). Further studies are needed in this direction.

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