

Commission internationale pour l'étude et  
l'amélioration de l'enseignement des mathématiques.  
www.cieaem.org

International commission for the study and  
improvement of mathematics education  
www.cieaem.org



## *Proceedings / Actes*

CIEAEM 71  
Braga (Portugal)  
July, 22 - 26 2019

# CONNECTIONS AND UNDERSTANDING IN MATHEMATICS EDUCATION: MAKING SENSE OF A COMPLEX WORLD

\*\*\*

# CONNEXIONS ET COMPREHENSION DANS L'ENSEIGNEMENT DES MATHÉMATIQUES: DONNER UN SENS A UN MONDE COMPLEXE



Editor: Benedetto Di Paola, Pedro Palhares  
Editor of the Journal : Benedetto Di Paola

**International Program Committee / Comité International de Programme:**

Alexandra Gomes - Portugal  
Ana Serradó – Espagne  
Andreas Moutsios-Rentzos - Grèce  
Ema Mamede - Portugal  
Fragkiskos Kalavasis - Grèce  
Gail FitzSimons - Australie  
Gilles Aldon - France  
Javier Díez-Palomar - Espagne  
Lisa Boistrup - Suède  
Marcelo Bairral - Brésil  
Monica Panero - Suisse  
Pedro Palhares - Portugal  
René Screve - Belgique

**Local Organizing Committee / Comité Organisateur Local:**

Alexandra Gomes  
Ema Mamede  
Filipa Balinha  
Joana Tinoco  
Leticia Martins  
Maria Helena Martinho  
Paula Cardoso  
Pedro Palhares  
Sara Ribeiro

## Index

<b>Information about CIEAEM 71 and presentation of the Volume/Informations sur la CIEAEM 71 et présentation du Volume</b>	p. 3
<b>PLENARIES</b>	p. 5
<i>Making sense of probability in professional and everyday life</i> Batanero C.	p. 7
<i>Facing complexity with simplicity</i> J. Gimenez	p. 19
<i>Indigenous knowledges re-evaluating mathematics and mathematics education</i> K. Owens	p. 29
<b>WORKING GROUP 1 / GROUP DE TRAVAIL 1</b>	p. 45
<i>WG1/GT1 Report: Learning in an increasingly complex world</i> G. FitzSimons, Alexandra Gomes	p. 47
<i>Paper folding for an active learning of mathematics: an experience with preservice teachers</i> I. Vale, A. Barbosa, I. Cabrita	p. 53
<i>Results of a pilot study on primary school students' intuitive perception of infinity</i> A. Tsampouraki, S. Kafoussi	p. 61
<i>Investigating the role of games in mathematics education</i> D. Ferreira	p. 69
<i>Exploring figure background perception of young children</i> F. Balinha, E. Mamede	p. 75
<i>Obstacles, initiatives and perspectives on mathematical teaching in engineering courses in Brazil: an overview based on studies published in COBENGE (2017-2018)</i> J. Aparecida Amorin Colombo, E. Zarpelon	p. 87
<i>Is it indispensable? The attitudes of students entering Athens Technical University toward complex numbers</i> K. Pavlopoulou, T. Patronis, Y. Kandias	p. 99
<i>Learning mathematics outside the classroom: experiences in primary school</i> L. Fonseca, J. Cacaïs, C. Fernandes	p. 105
<i>Learning strategies for problem solving using explicit and implicit teaching</i> M. Amit, Y. Portnov-Neeman	p. 115
<i>The interplay of creative problem solving and formal knowledge</i> O. Uziel, M. Amit	p. 121
<i>An introduction to the theory of complexity: A case study with dynamic systems and fractality</i> S.R. Sánchez	p. 127
<i>Design of situations fostering horizontal mathematization: benefits from epistemological analysis of experts' practices</i> S. Yvain-Prébiski, S. Modeste	p. 139
<i>Reading and constructing graphs: analysis of students' results based on national evidence</i> C. Spagnolo, S. Tiralongo	p. 151
<i>Promoting understanding through simulation games. An application to teaching geometry in primary school</i> A. Piu, and C. Fregola	p. 157
<b>WORKING GROUP 2 / GROUP DE TRAVAIL 2</b>	p. 169
<i>WG2/GT2 Report/Rapport: Mathematics teacher education/Éducation des enseignants en mathématiques</i> M. Panero, E. Mamede	p. 171
<i>Teacher Training Activities in Secondary School Focused on Interdisciplinarity</i> M.G. Adesso, R. Capone, C. Spagnolo	p. 177
<i>Supporting pre-service elementary teachers to promote students' mathematics learning with understanding: The use of tasks</i> A. Spiliopoulou, K. Ntouma, A. Boufi	p. 187
<i>Portuguese school teachers' mathematical knowledge for teaching: questions on numbers and operations</i> C. Vasconcelos Pereira Gonçalves; A. Gomes	p. 193

<i>Encounters with Mathematics – Reflection of the personal mathematical learning biography in teacher training</i>	
C.S. Günther, K. Reitz-Koncebovski, P. M. Klöpping	p. 199
<i>Formative assessment workshops as a tool to support pre-service teacher education on argumentation</i>	
C. Sabena, G. Albano, A. Pierrì	p. 205
<i>The training of teachers for the teaching of Mathematics through the creation of stories of Children's Literature</i>	
E. Souza de Alencar	p. 217
<i>Early algebra. Classroom practices</i>	
P. Bonisconi, M. Cazzola, G. Riva, E. Rottoli, S. Sorgato	p. 225
<i>Complex competencies in mathematics teacher education: Prospective teachers unpacking mathematics and mathematics teacher identities</i>	
L. DeAun Guzmán, C. Oropesa Anhalt, J. R. Sheldon	p. 235
<i>Evolution of preservice primary teachers' attitudes towards mathematics and its teaching</i>	
P. Di Martino, M. Panero, S. Sbaragli	p. 239
<i>Provocations in mathematics: Teachers' attitudes</i>	
S. Klymchuk, C. Sangwin	p. 247
<i>Didactical Suitability Criteria in Videos of Lesson Study</i>	
V. Hummes, A. Breda, A. Sánchez, V. Font	p. 257
<b>WORKING GROUP 3 / GROUP DE TRAVAIL 3</b>	p. 269
<i>WG3/GT3 Report: Teaching for connections and understanding/Enseigner pour les connexions et la compréhension</i>	
A. Moutsios-Rentzos, A. Serradó Bayés	p. 271
<i>Mental acts activated when making connections to understand the complexity of the stochastic thinking</i>	
A. Serradó Bayés	p. 277
<i>Appearances of the equals sign in primary school mathematics and natural sciences: an interdisciplinary, systemic approach</i>	
A. Moutsios-Rentzos, V. Pinnika, G. Kritikos, F. Kalavasis	p. 285
<i>Acting with the pantograph: potentials and obstacles</i>	
E. Koleza	p. 295
<i>Examining the core practices and routines to develop mathematical modeling competencies in early modelers</i>	
J. Suh	p. 307
<i>A problem-solving experience: The teacher's perspective</i>	
L.G. Martins, M.H. Martinho	p. 319
<i>Mathématiques et approche à la complexité par le coding: étude d'une pratique d'innovation au collège</i>	
M. Polo, F. Curreli, G. Deiana, A. Desogus	p. 325
<i>Comment reconnecter les rapports trigonométriques dans le triangle rectangle et dans le cercle trigonométrique? How to reconnect the trigonometric ratios in right triangles and in the unit circle?</i>	
M. Pierard, V. Henry	p. 337
<i>Complex tasks for learning in a complex world?!</i>	
N. Bohlmann, R. Benölken	p. 345
<i>Making Sense of Fractions at Primary School — the Case of Teacher João</i>	
P. Cardoso, E. Mamede	p. 355
<b>WORKING GROUP 4 / GROUP DE TRAVAIL 4</b>	p. 365
<i>WG4/GT4 Report/Rapport: Mathematics Education with Technology/Mathématiques avec les technologies</i>	
G. Aldon, I. Cabrit	p. 367
<i>Affordances of 360-degree video in helping pre-service educators notice young children engaging with mathematics</i>	
A. Cooke	p. 375
<i>Using WhatsApp to share mathematical ideas</i>	
B. Mota & Rosa A.T Ferreira	p. 383
<i>Graphic Programming Artefacts in the development of geometric skills</i>	
E. Cunha, I. Cabrita, L. Fonseca	p. 393

<i>School guidance: a game for middle and high school students</i>	
G. Aldon	p. 403
<i>Effect of a collaborative and iterative GeoGebra module on in-service mathematics secondary teachers' zones</i>	
H. El-Kasti	p. 413
<i>Touching on screen, exploring and making sense regarding quadrilaterals with FreeGeo App</i>	
Marcelo Bairral	p. 421
<b>WORKING GROUP 5 / GROUP DE TRAVAIL 5</b>	p. 427
<i>WG5/GT5 Report/Rapport: Connections with culture/Connexions avec la culture</i>	
L. Björklund Boistrup, G. Bini	p. 429
<i>Can Ethnomathematics Save Itself from its Colonialist Past? Yes, by Living Together with Mathematics Education</i>	
C. Stathopoulou, P. Appelbaum	p. 433
<i>Lecture de textes historiques dans la formation à l'enseignement des mathématiques : choix et modalités de lecture</i>	
D. Guillemette	p. 443
<i>Is this the real life? Connecting mathematics across cultures</i>	
G. Bini, O. Robutti	p. 455
<i>Engaging Children's Multiple Mathematical Knowledge Bases with Mathematical Modeling Tasks in Elementary Classrooms</i>	
J.M. Aguirre, C.O. Anhalt, E.E. Turner, M.Q. Foote, A. Roth McDuff	p. 463
<i>Exploiting the six mathematical activities for responding to a real community's problem with young Roma students</i>	
G. Kyriakopoulos, C. Stathopoulou	p. 472
<i>Challenging theory versus practice: Interfaces between mathematics and vocational education</i>	
L. Björklund Boistrup, V. Lindberg	p. 477
<i>The Predicament of Culture: The Dangers of Hoping that Culture Will Solve Problems of Mathematics Education</i>	
P. Appelbaum, C. Stathopoulou	p. 483
<i>Ethnomathematical study on folk dances: linking to school-mathematics</i>	
S. Ribeiro, P. Palhares, M. Jesús Salinas	p. 491
<i>Constructing students' mathematical identity at home: the case of homework</i>	
S. Kafoussi, A. Moutsios-Rentzos, P. Chaviaris	p. 499
<b>WORKSHOPS / ATELIERS</b>	p. 507
<i>Odyssey Mathematics: was Homer a mathematician?</i>	
A. Drivet	p. 509
<i>Translating practices for reflecting on ourselves: Lesson Study</i>	
C. Manolino, R. Minisola, O. Robutti, F. Arzarello	p. 519
<i>ResCo : un dispositif et des situations pour travailler la modélisation mathématique en classe. L'exemple d'un problème industriel d'optimisation de découpes de vitres</i>	
J. Lavolé, S. Modeste, S.Yvain-Prébiski, Groupe ResCo de l'IREM de Montpellier	p. 527
<i>Ethnomathematical study on folk dances: discussing a method for mathematical modelling of choreography</i>	
S. Ribeiro, P. Palhares, M.J. Salinas	p. 531
<b>FORUM OF IDEAS / FORUM AUX IDEES</b>	p. 539
<i>Connecting mathematics and sciences through children's literature. An experience with Pre-service Preschool Educators</i>	
A. Gomes, and A. S. Afonso	p. 541
<i>L'expérience des connexions – leur apport à la compréhension des maths au lycée.</i>	
C. Chevier	p. 547
<i>Problem-Solving Methodology applied to the teaching of Calculus</i>	
E.B. Azevedo, P.M.B. Palhares, E.B. Figueiredo	p. 555
<i>Exploiting symmetry for culturally meaningful mathematics with Roma students</i>	
G. Kyriakopoulos, I. Chronopoulou	p. 561

<i>Developing didactic-mathematical knowledge on proportionality in prospective elementary school teachers</i>	
M. Burgos, J. D. Godino, M. Rivas	p. 565
<i>School math and math in video games</i>	
M. Čujdiková	p. 571
<i>Changing students approach to studying calculus</i>	
M. Dagan, P. Satianov, M. Teicher	p. 575
<i>Technology as a tool to understand sampling in binomial distributions</i>	
N. Begué, C. Batanero, M.M. Gea, P. Beltrán-Pellicer	p. 579
<i>Scientific calculator as a valued partner in teaching calculus</i>	
P. Satianov and Miriam Dagan	p. 585
<i>Mathematics and Living together : Social Process and Didactic Principle</i>	
Y. Aberkane	p. 589

## **Information about CIEAEM 71 and presentation of the Volume** **Informations sur la CIEAEM 71 et présentation du Volume**

The 71st CIEAEM conference was held from 22nd to 26th July 2019 at the Instituto de Educação da Universidade do Minho, Campus de Gualtar, Braga. Several participants from many countries all over the world, actively participated to the Conference. The Theme “*Connections and understanding in mathematics education: Making sense of a complex world*” was really appreciated by participants; researchers, teachers, educators, and students worked together in the true of CIEAEM spirit, with a collaborative and inspiring attitude.

*How can we re-conceptualise learning with understanding in a complex world?, What kind of mathematics training should teachers have in order to be able to promote learning with understanding?, In relation to connections and understanding, what kind of teaching methods are more appropriate?, How do we evaluate and/or research the resources from the perspective of the connections and the understanding they try to promote?, How can ICTs contribute to learning rich in connections, in an increasingly complex world?, How can ICT be used in teacher training to promote understanding in mathematics?, Is it possible to understand peoples’ lives from an ethnomathematics perspective?, How can school mathematics take into account the culture developed by young people in their everyday lives?, How to take advantage of cultural aspects to enrich the teaching and learning of mathematics?*

These are only some of important questions on which all the participants had the possibility to fruitfully discuss in critical and constructive ways.

Many of the Conference participants were also authors of significant papers presented during the working groups, workshops, and poster (forum of ideas) activities. This volume contains all final versions of these papers.

We thank all the contributors and all the participants to the conference and we are grateful to Pedro Palhares and all the members of the International Programme Committee and the Local Organizing Committee for the realization of the 71st CIEAEM conference. Particularly we want to thank the Working Group animators, who, as in previous CIEAEM meetings, worked in a well high-quality way.

March, 2020

Benedetto Di Paola,  
Pedro Palhares





# Translating practices for reflecting on ourselves: Lesson Study

**Carola Manolino, Riccardo Minisola, Ornella Robutti, and Ferdinando Arzarello**

*Università di Torino*

E-mail: [carola.manolino@unito.it](mailto:carola.manolino@unito.it), [riccardo.minisola@unito.it](mailto:riccardo.minisola@unito.it)

**Abstract.** Lesson Study (LS) is a collaborative methodology for teachers' professional development (TPD) rooted in Japan. In LS, a group of teachers and experts collaborates to the detailed planning of a one-hour lesson. The difference between LS and other methodologies is the collaborative foundation of the experience, there is no evaluation on the performance of a single member of the group. We believe that, in the Italian context, LS can be an appropriate tool to answer in an efficient way the Ministry's demands for a "mandatory, permanent and strategic" TPD and for the "establishment of adequate networks for professional collaboration", while maintaining the focus on teachers' needs. Discussing, observing and reflecting on your own and others' practices can help in re-thinking your own professionalism while relating to a community of peers. In the workshop we will work on one of the main tools in LS, the Lesson Plan, and discuss how the encounter of such a foreign tool can lead to self-reflection on one's own practices.

**Abstract.** Le Lesson Study (LS) est une méthodologie collaborative pour le développement professionnel des enseignants, enracinée en Japon. Dans la méthodologie LS, un groupe d'enseignants et d'experts collabore à la planification détaillée d'une leçon d'une heure. La différence entre la LS et les autres méthodologies est la base collaborative de l'expérience, il n'y a pas d'évaluation sur la performance d'un seul membre du groupe. Nous croyons que, dans le contexte italien, la LS pourrait être un outil approprié pour répondre de manière efficace aux demandes du Ministère pour un développement professionnel des enseignants "obligatoire, permanente et stratégique" et pour "l'établissement de réseaux adéquats de collaboration professionnelle", tout en maintenant une focalisation sur les besoins des enseignants. Discuter, observer et réfléchir sur ses propres pratiques et sur celles des autres peut aider à repenser son propre professionnalisme tout en établissant des relations avec une communauté de pairs. Au cours de cet atelier, nous travaillerons sur l'un des principaux outils du LS, le Lesson Plan, et discuterons de la façon dont la rencontre d'un tel outil étranger peut mener à une réflexion personnelle sur ses propres pratiques.

## 1. Lesson Study

Lesson Study (LS) is a collaborative methodology for teachers' professional development (TPD) rooted in Japan. Since 1999, researchers in TPD and didactics from all over the world have started studying the methodology (Huang & Shimizu, 2016), and since 2003 the Asia-Pacific Economic Cooperation (APEC) has been following its international diffusion. Catherine Lewis, vice-president of WALs, has had an essential role in the world-wide diffusion of LS (Bartolini Bussi & Ramploud, 2018).

In a LS, a group of at least three teachers (which we will call Lesson Study Group; it can include one or more student teachers or university experts) collaborates to the detailed planning of a one-hour lesson, to be taught in one of the teachers' classroom observed by the other teachers, and discussed by the group. The difference between LS and other methodologies is the collaborative

foundation of the experience, which leads to the establishment of a sense of diffused responsibility between the members of the group. Moreover, the observation of the lesson indicates LS as a form of action-research. There is no evaluation on the performance of a single member of the group: the focus is on the lesson and the students, not on teachers' individual ability.

The National strategies for Teachers' Professional Development document of the Italian Ministry of Public Education, covering the three-year period 2016 – 2019, stresses the importance of addressing issues such as: teachers' isolation in managing pupils' learning; connecting work and professional development; difficulties in applying in a real-classroom context the didactic innovations proposed by universities. We believe that LS can support the researchers' and teachers' communities in answering the Ministry's demands for a “mandatory, permanent and strategic” TPD and for the “establishment of adequate networks for professional collaboration”, while maintaining the TPD focus on teachers' needs. Discussing, observing and reflecting on your own and others' practices can help in re-thinking your own professionalism while relating to a community of peers. The encounter with others, from this point of view, is one's self-rediscovery (Mellone, Ramploud, Di Paola, & Martignone, 2018).

## **2. Italian Lesson Study: ideas from Turin**

As shown from the literature review (Fernandez & Yoshida, 2004; Minisola, 2016; Robutti, et al., 2016), LS is generally a three-step cycle aimed at creating a virtuous process in which teachers can grow continuously (Ramploud & Munarini Frenesi, 2015). In its (cultural) transposition to Western cultures – particularly the Italian one – these steps can be defined the “essentials” of LS: establishment of long-term learning goals and lesson planning, implementation and observation of the research lesson, discussion on the lesson. These steps can be repeated, like a life cycle in which each lesson is the foundation for subsequent growth. In the Italian context, the time expected for each step is: at least 2 hours for goals establishing, 2 hours for lesson planning, 1 hour for the lesson, 2 to 4 hours for the discussion. The overall commitment for teachers is predicted in 7 to 10 hours.

Initial findings mandate to clarify that, in LS, a lesson is a specific moment in the classroom routine (i.e. the mathematics lesson in class 3B held from 9 to 10 a.m. on February the 3rd, 2019). The group of lessons dedicated to a specific topic (i.e. continued fractions) is called teaching unit.

The existence of a LS-Group is tied to that of the observed lesson. Even so, the same group of people can participate in more study cycle and establish a stable-over-time community of practice. The aim is to build and institutionalize a collaborative methodology, which can sustain teachers in both their job and professional development, focusing on the new multicultural context we all are living in.

The tentative structure of Lesson Study in the Italian context is:

*Definition of long-term educational objectives:* LS is a form of action-research, in which teachers collaborate to improve their professionalism in accordance to the context in which they work. The reasons to engage in LS might stem from different teachers' needs: i.e., difficulties in confronting with certain mathematical topics, improving strategies to involve students, experimenting new didactical methodologies. A research question is formulated by the group in accordance to these needs, and exploring the possible answers is the objective of the LS. Moreover, Italian secondary school teachers have autonomy on defining the educational plan, referring to Indicazioni Nazionali – the national curriculum by Ministry of Education – containing knowledge and competences related to the specific kind of school. Moreover, each school has its Piano Triennale dell'Offerta Formativa or PTOF (“three-year plan of the educational offer”), in which more specific educational objectives are described. Thus, in the first phase of LS in Italy, the teachers choose a teaching unit and the related long-term learning objectives, in accordance to Indicazioni Nazionali and PTOF: these objectives should be relevant to the whole group (i.e. because they are difficult to attain) to

promote engagement, and related to the research question(s). One (or two) demonstrating teachers should be chosen, to develop a lesson aimed at a specific context, and to investigate the answer to specific (and yet shared by the group) needs.

*Lesson Planning:* The demonstrating teacher(s) prepares – on his/her own or working with colleagues – a draft of the Lesson Plan, describing: the class context (such as the general level of knowledge and competencies, or the presence of students with special educational needs: the Italian school is inclusive, meaning that in Italy there are no special schools for students with learning difficulties, physical disabilities or behavioural problems); the teaching unit in which the lesson is inscribed; a proposal for the 1-hour lesson in accordance – as much as possible – to the class' didactic contract. The tentative Lesson Plan is given to the whole LS-Group before the planning meeting, in which the group discusses the details and decides: the phases of the lesson, the time to allocate for each phase, the teacher's requests to students, how the teacher should react to some students' expected reactions, what are the educational aim of each phase, which classroom grouping strategies to apply. The plan is carefully fitted on both the classroom's pupils and the demonstrating teacher's disposition. The group proposes ideas, techniques, strategies, but ultimately it is the demonstrating teachers' choice what to implement and what is not doable in his/her classroom. Observational focuses are established in accordance to the initial aims and to the group's decisions in planning the lesson: i.e. the group might decide to focus on the efficacy of artefacts proposed by the teacher to the class, of the grouping strategies, of the problem structure, etc. The group may decide to use a table to guide the observation using some learning descriptors, and/or to focus on some students considered representative of the classroom situation. In this phase, appointing a secretary to record the discussion, and a moderator to the discussion, is useful in terms of time management.

*Lesson implementation and observation:* The teacher and the observers enter into the classroom to teach and observe the prepared lesson. A series of preliminary encounters might be necessary to get students accustomed to the presence of other people. The observers are silent and should not influence the class' practice. The presence of all the members of the LS-Group is not necessary, but a video record of the lesson is recommended.

*Discussion:* LS methodology is focused on the efficacy of the prepared lesson in accordance to the established objectives, not on the ability of the individual teacher. Before the discussion meeting, the whole LS-Group (including the teacher) has shared and studied the observers' reports and possible videos. The discussion is opened by the teacher, who shares his/her impressions and observations on what occurred in the classroom. The whole group discusses how to fix what did not work, improve what did, reflect on how to deal with (and consider the possibility of) the unexpected: it is not possible to plan for every instance that may occur. Missing something is a "mistake" of the whole group, even if the teacher in charge of the lesson was the one who had to respond to the unexpected event(s). As different teachers make different kind of expertise available for the group, this is the opportunity for both the demonstrating teacher and the whole group to learn how to manage unexpected situations or improve non-optimal behaviours, absorbing new ideas from others' experience. This discussion may or may not result in a new "improved" Lesson Plan, which can be taught by the same or another teacher in a different classroom, bringing about a new study cycle. As in the planning phase, choosing a secretary and a moderator is advised.

### **3. Plan for the workshop**

The workshop will be organised in the following steps:

1. A 10 minutes introduction on Lesson Study and the Lesson Plan
2. The participants will be divided in small groups of 3-4 persons each. The groups should be organised as much as possible according to nationality and school level. Each group will have about 10 minutes to decide, possibly according to the participants' training needs, which activity they will work on among the proposed ones (differentiated by school level).

3. 60 minutes will be dedicated to the lesson planning, using the empty Lesson Plan form provided. Each group is asked to keep track of the encountered difficulties, whether they are educational, planning-related, organizational, and also relationship-related difficulties. Since the proposed Lesson Plan structure is adequate to the Italian context, the focus will be especially on those related to the different cultural context the participants are used to.
4. A 10 minutes meta-discussion on the activity: what are the most striking differences between this planning methodology and those used in the participants' usual contexts? Are there any analogies? Does the encounter with the Lesson Plan, a tool coming from a different cultural and institutional context, bring a reflection on the participants' own practices?

#### **4. The Workshop**

The workshop lasted 70' and the tentative times were adjusted accordingly: 7' for the introduction, 8' for deciding the teaching material, 45' for lesson planning, 10' for the discussion. 17 people attended the workshop, from a number of nationalities, ages and professional backgrounds. In particular, 7 of them were university students and declared no teaching experience. Audio recording the session was not possible due to GDPR limitations; the presenters noted arguments and comments on a notebook.

During step 1, material on Lesson Study was presented to the attendants. The presentation focused on Lesson Study in relation with the Japanese context, analysed the similarities with the Italian context, and the peculiarities of the latter (i.e.: both contexts focus on the pupils and design educational plans in terms of long-term goals; Japanese teachers mainly work inside schools, Italian ones mainly work at home; Italian school is inclusive, Japanese school is not). A possible adaptation of LS in the Italian context was proposed, and a copy of an empty Lesson Plan (cfr. Appendix) was distributed to each participant. An overview of the proposed structure for the Lesson Plan was discussed, and the participants were proposed the task of the workshop for step 2. The groups resulting by the participants self-organisations were more numerous than expected and heterogeneous, differently from what asked by the presenters. Rearranging them was deemed not necessary. In the case that some of the participants needed support in planning for a lesson, two teaching activities were proposed on different mathematical topics: Heron's problem and its generalization for high school, exploration on non-planar surfaces for middle school.

At the beginning of step 3, the participants were asked to work and discuss within their groups, and three questions were presented to guide the work: (1) What are the most striking differences between this planning methodology and those used in the participants' usual contexts? (2) Are there any analogies? (3) Does the encounter with the Lesson Plan, a tool coming from a different cultural and institutional context, bring a reflection on the participants' own practices? About one third of the participants continued working on the teaching activities; after other 10' the presenter decided to ask to concentrate on the provided Lesson Plan, as it was the focus of the workshop.

By the beginning of the whole-group discussion (step 4), no one had compiled a Lesson Plan: when asked for a reason, a participant commented "analysing and discussing the Lesson Plan is more interesting, as we didn't have enough time to study the material", at which the others agreed. Three participants expressed curiosity about the structure of the lesson proposed in the Lesson Plan, and the presenters explained that it was inspired to Calvani (2014) and Bartolini Bussi et al. (2017).

Participants from France and Spain noted that the philosophy of the Lesson Plan is not far from the work teachers usually do in their context. In Spain, for example, "our teachers do this kind of work for each lesson, and lesson plans are uploaded online for families and others to be consulted"; others from Holland and Switzerland explained that detailed lesson planning is usually mandatory for pre-service teachers during their training but not for in-service ones, and it is usually kept as internal documentation for the school. Two participants from Poland commented that "[detailed

lesson planning] is unusual for both prospective and in-service [teachers], especially with all the details proposed in this Lesson Plan". Spain followed that "estimating the time for each phase, we don't do that" and suggested a modification in the table to highlight this characteristic of the proposed Lesson Plan.

All the participants agreed that specifying the "educational intentionality" for each phase was "the real peculiarity of this document". When asked for clarification, one young participant from Poland explained: "I believe that this is very important when doing pre-service training. It makes you aware that everything you do in classroom is relevant for your students". A participant from Belgium agreed: "it is something you tend to forget even when you are an experienced teacher, so it would be nice to have a reminder every now and then. It makes you aware". The participant from France concluded: "detailed lesson planning is very difficult no matter the subject, but I believe that in Mathematics it is especially important: Mathematics gives students critical thinking, develops their cognitive abilities, is the basis for their scientific approach. We all need to be very careful when we teach it, we are shaping the future... and we need all the help we can get".

## **5. Discussion**

Designing a detailed lesson plan is no easy task, as proven by the fact that no participant produced a complete Lesson Plan, not even those accustomed to detailed documentational work. Furthermore, the task of planning and compiling the plan for a lesson depends on the documentational work that may vary from context to context, for cultural and institutional reasons.

The proposed Lesson Plan is the result of the reflection on the Italian culture, institutional context, usual practices. It embeds Italian institutional peculiarities, such as the different stance on educational objectives: namely, long-term objectives are presented in the National Guidelines, whereas lesson objectives are decided by each teacher individually. A Lesson Plan fitted to a certain context is not immediately effective: its conscious use requires familiarity with the mathematical knowledge, curriculum, teaching traditions, institutional context.

In this sense, we might say that the workshop evolved unexpectedly. The focus shifted from the analysis of Lesson Plan's specific steps to its general issue as a design tool in a school. This new focus nourished the participants' confrontation. The self-organized heterogeneity enriched the discussion, albeit some realism got lost in the transition (namely, no Lesson Plan was compiled), and answered to the needs of the participants.

In conclusion, preparing and studying a detailed plan for an effective Mathematics lesson is perceived both like a challenge and a necessity. Discussing and sharing educational experiences, provided they happen within customary practices of designing and programming, might sustain to collaboratively overcome the perceived challenges of teachers' professionalism. To achieve this, it is necessary that researchers in mathematics education deepen their studies of interaction with teachers in order to improve the collaboration with them in concrete teaching activities.

## **Appendix**

In this appendix, the empty Lesson Plan used in the workshop is presented. Please note that this is a condensed version. The printed version provides enough space to write. This version of the Lesson Plan is inspired by Bartolini Bussi et al. (2017) and Fernandez & Yoshida (2004).





**Lesson Plan**

School:  
Class:

<p><b>Description of the class</b> <i>(classroom composition, prevalent teaching methodologies)</i></p> <p><b>Context</b> <i>(learning trajectory in which Lesson Study is held):</i></p> <p><b>Educational goals on competences</b> <i>(in accordance to the National Programme)</i></p> <p><b>Specific learning goals</b> <i>(in accordance to the National Programme)</i></p> <p><b>Initial situation of the class</b> <i>(with respect to mathematical competences: goals already achieved, pre-requisites for the lesson)</i></p> <p><b>Organization of the didactic work</b> <i>(the total duration of the project; the place where it is carried out)</i></p> <p><b>Organization of the teaching unit:</b> LESSON 1: LESSON 2: LESSON ...N:</p> <p><b>Methods of evaluation</b> <i>(how it is intended to evaluate the degree of achievement of the goals, how it is intended to evaluate the functioning of the activity on the class group)</i></p>
<b>RESEARCH LESSON PLAN</b>
<p><b>Topic/content of the lesson in question</b> <i>(title)</i></p>



<p><b>Goal(s) of the lesson</b> <i>(Learning goals based on National Programme; punctual and minimum goal of 1h of lesson)</i></p>					
<p><b>What is the purpose of the observation?</b> <i>(observational focus)</i></p>					
<b>Presentation of the lesson (mathematics)</b>					
<b>Description of the activity</b>	<b>Task and/or teacher's questions</b>	<b>Student reactions and directions for the teacher</b>	<b>Grouping</b>	<b>Time table</b>	<b>Educational Intentionality (the reasons for the choices)</b>
Introduction to the lesson and presentation of the topic	<i>(summary to the class, by the teacher, of the activities already carried out and specific of the topic of the day)</i>		<input type="checkbox"/> Whole class <input type="checkbox"/> Small group <input type="checkbox"/> Pairs <input type="checkbox"/> Individually		
Homework check (optional)			(where appropriate, a list of the groups and their reasons for doing so)		(Explanations of the problematics to be highlighted)
Formulation/presentation of the problem of the day					
Presentation / clarification of the problem of the day					
<b>Working on the problem</b>					
<b>Description of the activity</b>	<b>Task and/or teacher's questions</b>	<b>Student reactions and directions for the teacher</b>	<b>Grouping</b>	<b>Time table</b>	<b>Educational Intentionality (the reasons for the choices)</b>
Working on the sub-problem (optional)	<i>(if the task is complex, it is possible, intentionally, to break down the problem into simpler units)</i>		<input type="checkbox"/> Whole class <input type="checkbox"/> Small group <input type="checkbox"/> Pairs <input type="checkbox"/> Individually		
Working on the problem	<i>(criteria for group composition, methodologies)</i>		(where appropriate, a list of the groups and their reasons)		(reasons for the choice of activities, materials and methods)

**References**

Bartolini Bussi, M. G., & Ramploud, A. (2018). *Il lesson study per la formazione degli insegnanti*. Roma: Carocci.

Bartolini Bussi, M. G., Bertolini, C., Ramploud, A., & Sun, X. (2017). Cultural transposition of Chinese lesson study to Italy: An exploratory study on fractions in a fourth-grade classroom.

*International Journal for Lesson and Learning Studies*, 6(4), 380-395. doi: 10.1108/IJLLS-12-2016-0057.

Calvani, A. (2014). *Come fare una lezione efficace*. Roma: Carocci

Fernandez, C., & Yoshida, M. (2004). *Lesson Study: A Japanese approach to improving mathematics teaching and learning*. Mahwah: Lawrence Erlbaum Associates.

Huang, R., & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher developers, and linking theory and practice through lesson study in mathematics: an international perspective. *ZDM* 48(4), 393-409. Springer.

Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM*, 51(1), 199-212.

Minisola, R. (2016). *Insegnanti di matematica che lavorano in collaborazione: panoramica internazionale e contesto italiano*. Master's Dissertation. Università di Torino. [https://www.researchgate.net/publication/316669730\\_Insegnanti\\_di\\_matematica\\_che\\_lavorano\\_in\\_collaborazione\\_panoramica\\_internazionale\\_e\\_contesto\\_italiano](https://www.researchgate.net/publication/316669730_Insegnanti_di_matematica_che_lavorano_in_collaborazione_panoramica_internazionale_e_contesto_italiano). Accessed 29 September 2019.

Ramploud, A., & Munarini Frenesi, R. (2015, Giugno 10). Il "Lesson Study" [guanmo ke]: trasposizione culturale di una metodologia di formazione. *Scuola Italiana Moderna*, 10, 54-61.

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM*, 48(5), 651-690.

