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ABOUT THESE PROCEEDINGS

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AUDIOFUNCTIONS 2.0: A MULTIMODAL ENVIRONMENT FOR INCLUSIVE MATHEMATICS EDUCATION

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ABSTRACT

This poster concerns a dynamic and multimodal interactive digital artifact for representing graphs of real-valued functions of a real variable: Audiofunctions[16]. This artifact operates both at a visual level, displaying a function graph and a moving point on it, and at an auditory level, sonifying the coordinates of the moving point and responding dynamically to changes in the independent variable as the cursor moves. The study introduces its version 2.0, in which different kinds of sonification and auditory cues for undefined or off-screen function values have been provided, ensuring simultaneous, non-conflicting perception of multiple features. It also discusses sustainability implications arising from the inclusive use of Audiofunctions, highlighting the role of sonification and dynamic responsiveness in the study of function graphs and providing insights into the potential impact of inclusive digital tools on interactive learning experiences.

Keywords: Function graphs, multimodality, inclusion, sonification, visual impairments.

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[16] This artifact, originally developed by the Laboratory "S. Polin" at the University of Turin, is currently being researched within the Erasmus+ Project "Sonairgraph - SONification for Accessible and Inclusive Representation of GRAPHs in Education" (https://sonairgraph.unito.it/), in response to Call 2024 Round 1 KA2-KA220-HED - Cooperation partnerships in higher education, CUP B63C24000960006. The current version of the software is available at: https://audiofunctions-plus.netlify.app/

Audiofunctions 2.0: A multimodal environment for inclusive mathematics education

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Introduction

Since 2014, the Laboratory "S. Polin" at the University of Turin has been developing and researching **Audiofunctions** (**AF**), a **software** originally conceived as a technical response to the challenge of **making function graphs**—an inherently visual mathematical representation—**accessible to blind users** already proficient in mathematics, such as university students and professionals. Previous development of Audiofunctions focused mainly on accessibility and sustainability issues (Bernareggi et al., 2016; Ahmetovic et al., 2019). Results from these studies indicate that AF offers an efficient alternative to tactile graphics through dynamic and multimodal interaction, **using auditory feedback** and **proprioceptive techniques** to create a multimodal learning/interacting experience. Nevertheless, the assumption that technical access—however sophisticated—would automatically ensure access to mathematical meaning proved limiting. This motivated a shift towards studying users' meaning-making processes, particularly with expert blind users, to understand how they conceptualize function graphs through the auditory channel. As an ongoing research project, the development of the software is now integrating these educational concerns to ensure AF meets the highest standards, also in an inclusive and pedagogical perspective. AF is currently being studied as part of the Erasmus+ project **SONAIRGRAPH**. Here we present the Audiofunctions 2.0 implementation choices organized into macro areas, and for each one we briefly outline what has changed and why compared to the first Audiofunctions.web version.

Audiofunctions 2.0 Design

Interaction model

The AF web interface is designed according to the Web Content Accessibility Guidelines (WCAG); thus, the user interface elements and the interaction modalities are accessible both through mainstream peripherals (i.e. mouse and screen) and different assistive technologies (e.g., screen readers and Braille displays). Function graph can be explored through on-screen rendering and a combination of sonification and, if desired by the user, speech messages. The graph is displayed in a specific portion of the window, called the ***Graph Area***. Within this area, users can explore the function graph using the keyboard letter 'b' (**mode a**), keyboard left/right arrow keys (**mode b**), and the mouse (**mode c**) – see figure 1.

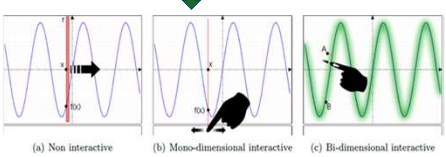

Figure 1. Function exploration modes (Bernareggi et al., 2016, p. 8)

Implementation criterias

Table 1 summarizes a comparison between AF versions. Two new **sound timbres** are available: a clarinet and an acoustic guitar. Once the timbre is selected, within the displayed *x* and *y* function range, we convey the current position of *x* in the interval and the function value: $y = f(x)$. We map the *x*-range to auditory **panning**: left *x*-values play louder in the left ear, right *x*-values in the right ear. The function value is mapped to the **pitch**, which directly maps the sound frequency to the function value. We map the *y*-range (minimum to maximum) to the instrument's pitch range (lowest to highest). We also added **pink noise** background to indicate when the current function value is negative – see figure 2.

Chart feature	Sound feature	Annotations
y-axis value	Pitch	Same as AF.web
Distance of mouse pointer from function curve	Loudness	Same as AF.web
x-axis value	Panning	Same as AF.web
Visualised chart bounds	Earcon 1	New in AF 2.0
Function not visible in the y range set for a given x	Earcon 2	New in AF 2.0
y-axis intercept	Earcon 3	New in AF 2.0
y negative quadrants	Pink noise	New in AF 2.0

Table 1. Audiofunctions features

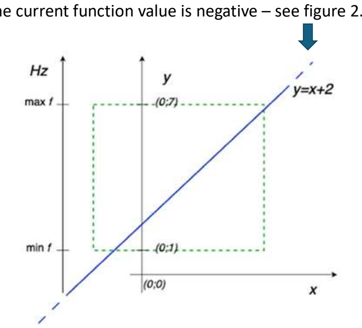

Figure 2. The pitch range adapts to the visualised *y*-axis range

Sonification

The foundational technique at the core of AF is **sonification.** In essence, sonification seeks to leverage auditory perceptual abilities to make data relationships understandable by representing them through sound (a data-dependent generation of sound). As a valuable technique for auditory visualisation, it enhances the interpretation and communication of information via non-speech audio representations.

Sonification:
"the technique of rendering sound in response to data and interactions" (Hermann et al., 2011, p. 1).

Continuous vs Discrete mode

AF 2.0 implements two types of sonification: continuous with the clarinet, and discrete with the acoustic guitar, producing effects of continuous pitch variation compared to the production of musical notes. Comparing continuous and discrete sonifications, two key differences are detected: the **pitch density** and the **threshold passing**.

Pitch density: The pitch density follows the same principle through a process known as **quantisation**, i.e. "the process that allows one to pass from infinite precision to finite precision in numerical representation" (Malcangi, 2008, p. 59, our translation) – see figure 3.

Threshold passing: it is a sonification strategy that leverages the lower pitch density in discrete sonification. In this strategy, instead of sonifying a value on the y-axis, we sonify the threshold crossing from one pitch to another – see figure 4.

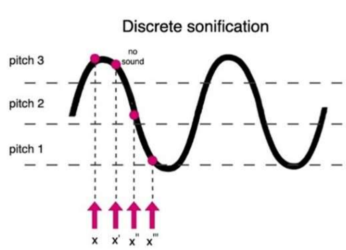

Figure 4. The discrete sonification

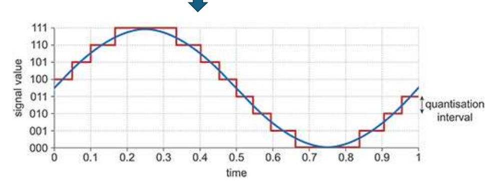

Figure 3. A sine wave (blu line) and one of its possible quantisation (red line) (Jones et al., 2020, p. 26)

Conclusions and future directions

AF 2.0 addresses nearly all limitations of previous version, moving beyond technical accessibility to embrace cognitive and educational perspectives. Users can **compare function rates** via discrete "speed" cues and **gain an embodied sense of derivatives** through paired-note auditory patterns. AF 2.0 provides auditory cues for undefined or off-screen function values, and no longer requires disabling screen readers, ensuring simultaneous, non-conflicting perception of multiple features. Initial interviews with blind expert users suggest **promising levels of graph comprehension** (Manolino et al., in press). Future work will focus on how students construct mathematical meaning through AF, particularly in inclusive settings. Insights from these studies will guide pedagogical applications and refinements of software design.

References

Ahmetovic et al., 2019
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Hermann et al., 2011
Jones et al., 2020
Malcangi, 2008
Manolino et al., in press

QR CODE

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