Financial bubbles and behavioral "Tatonnement" dynamics

Massimiliano Ferrara[a] **and Tiziana Ciano**[b], [a] Department of Law, Economics and Human Sciences, University Mediterranea of Reggio Calabria, Reggio Calabria, Italy; and [b] Department of Economics and Political Science, University of Aosta Valley, Saint Christophe, Italy

Key points

- Financial bubbles are sudden, rapid price increases in financial assets that are not justified by economic fundamentals.
- Speculative bubbles, resulting from anomalous price movements, occur when investors incorrectly evaluate asset prices, causing a significant increase in the real economy, despite the theory of efficient markets.
- This work demonstrates that financial bubbles can burst at any time and without restrictions, resulting from shifts in extrapolative expectations and neglect of extreme risks.
- Representative dynamics cannot be negative due to free transfer, and bubbles can persist and be distorted.

Abstract

This study shows that Financial Bubbles can be found without any limitations at any moment, leading to a deep variety of ramifications. Due to the high risks and unrealistic expectations, they can occur at any time. Depending on the state of transversality, the representative dynamics can be persistent or negative. Classic rational bubbles arise from the efficient market hypothesis, while representative bubbles can arise from psychological factors and exogenous shocks. Representative bubbles cause booms and busts, with fluctuations steeper than rational ones, and arise from distorted expectations caused by agents' inability to rationally understand signals.

Introduction

The issue of Financial Bubbles is often the subject of discussion and controversy. However, they are difficult to understand and are almost never precisely characterized, particularly in ex ante terms. The Financial Bubble is an upward movement of any financial asset until it reaches a constant and rapid increase in the price of an "asset" not justified by economic "fundamentals" often strictly connected with real economy. According to the theory of efficient markets developed toward the end of the 1960s, the prices of securities are determined by all the information available about them. Based on this theory, all financial securities are always valued correctly, based on publicly known information. As anticipated, Speculative Bubbles occur as a result of anomalous price movements, which cannot be justified by economic fundamentals and therefore in periods in which investors take an incorrect evaluation of the price of an asset as a reference. This situation can continue for months or even years while the bubble continues to inflate in the meantime. Therefore, the only way to hope to identify a bubble is to have a clear understanding of the phases that lead to its formation first and then to its explosion:

1. At the beginning, many investors begin to have confidence in the possibility of an increase in the value or price of an asset;
2. Therefore, the demands on that asset increase and the price also increases, according to the normal law of supply and demand, until it reaches very high levels;
3. At a certain point, an event occurs that casts doubt on whether the price can continue to rise;
4. The transition from euphoria to "distrust" then occurs. The number of those intending to buy drops and the attempts of those who want to sell and "get rid of" the asset in the shortest possible time increase;

5. The first drop in price arrives and the consequent acceleration of the pace of sales;
6. Finally, there is a definitive, often almost total, collapse of the price of that asset and the conclusion of the bubble.

The first bubble to burst was the tulip bubble which occurred in the 17th century in the Netherlands. At the time, tulip bulbs were a status symbol and were treated as collectors’ items. Furthermore, the flowers they produced were not monochromatic as today, but multicolored, thanks to the cultivation of varieties imported to Europe from exotic lands. Demand grew so rapidly that a single bulb of a specific type of tulip could cost as much as a thousand Dutch guilders. The strong demand that had pushed prices to unsustainable levels ran out in a few years and prices ended up collapsing, causing all the families who had gone into debt to buy the much-desired bulbs to fall into bankruptcy.

In the 18th century, another of the most famous bubbles occurred, the so-called South Sea bubble. The instrument involved was the shares of the South Sea Company, a British company which had been granted a monopoly for the international slave trade with the Spanish colonies in South America and the West Indies. The limited supply of contracts for the slave trade from Africa was the cause of the strong demand that drove the stock up, but the bubble lasted only a short time before bursting. Over time, many other bubbles have followed, such as the one that caused the Great Depression in 1929, the real estate bubble in Japan in the 1980s, the dotcom bubble and the subprime mortgage bubble, which triggered the global financial crisis (see Sornette and Woodard, 2010). Among the most recent bubbles is the one that occurred last year with the so-called meme stocks. Thanks to the power of social networks, a true collective euphoria has been created regarding the stocks of certain companies, called meme stocks. Several groups of retail investors organized themselves virally on various Internet platforms to push up the prices of low-float companies in which hedge funds held significant short positions to cause heavy losses to professional speculative investors. A company like Gamestop, which at the end of 2020 had a modest capitalization ($400 million), saw its value increase 80 times in the space of a few months without any fundamental justification. The capital held in customer accounts was used to pay other customers according to the most classic Ponzi scheme, but also to cover large expenses such as purchases of villas, donations to politicians and so on. In Galbraith (1994) a complete history of bubbles over the centuries is presented.

Literature review

After the events that followed the summer of 2007, understanding of financial bubbles began to falter. Economists have looked at rational models of domestic bubbles to understand why they have not monitored distortive price dynamics in, for example, real estate markets. It is therefore crucial to understand which type of model can best detect the dynamics of bubbles and how to enhance the rational ones. DeMarzo et al. (2008) in their work highlight the possibility that issues relating to relative wealth can play an important role in explaining financial bubbles by presenting a rational general equilibrium model. According to Diba and Grossman (1987) (who followed Tirole’s (1982) findings), a rational bubble can only increase if it exists at the time of the bubbly asset’s first trade; if it doesn’t burst at that point, it won’t rise at any other time. Furthermore, Giglio et al. (2016) examined the possibility of rational bubbles in the housing market prior to the 2007 turbulence. They discovered that the transversality requirement had not failed, ruling out rational bubbles as the cause of the overpriced homes.

Furthermore, by accounting for a behavioral component that is overlooked in the classic rational case, Strati (2017) defined a behavioral model of the bubble that can take into account over-optimism (or over-pessimism) in the market and detect a growing bubble even in situations where the transversality condition is met, or when the intertemporal equilibrium of prices is met.

One of the main ideas of Strati (2017) is that an exogenous influx of good news might produce a bubble by overheating the viewpoint of a so-called local thinker who is overly sensitive to incoming information. We seek to elucidate the formal origins of this type of behavioral bubble, known as the representative bubble (Kahneman and Tversky, 1972; Tversky and Kahneman, 1971, 1974, 1983). In financial economics, this refers to the application of non-Gaussian processes such as the Ornstein-Uhlenbec, which aim to address related issues (see, for instance, Issaka and SenGupta (2017)).

The expectations that arise from this heuristic are referred to as diagnostic because these positive shocks, or displacements (Kindleberger, 1978), increase the likelihood of an expected growth state. This is a diagnostic signal of their representativeness. Because diagnostic expectations are predicated on a kernel of truth, they exaggerate the direction of the path taken by the rational ones without altering it. Put another way, the representative expected state inflates with respect to the rational expectations component. Because these behavioral expectations are futuristic rather than adaptive, they are shielded from Lucas’ criticism. Kahneman and Tversky (1972) have studied this type of expectancy heuristic, and Gennaioli and Shleifer (2010) have formalized it most recently. Here, we adopt the so-called representativeness-based discounting approach, which is employed in Bordalo et al. (2018) and Strati (2017). The representativeness heuristic, however, can be defined in a number of ways (see Bordalo et al., 2016); additionally (Gennaioli and Shleifer, 2010), provides an application to financial fragility in which the representativeness heuristic entirely ignores downside risks. Ferrara et al. (2017) in their work formalize the birth of representative bubbles and their emergency conditions, discovering that they can start at any time based on the definition of the behavioral components. This contradicts the classical rational theory of bubbles, which relies on the transversality condition for bubbles to burst. Ferrara et al. (2018) present a representative bubble model with infinite agents, comparing it with the classical rational model. Discusses the role of the transversality condition in rational models, its necessity when agents are non-rational and how it triggers effervescent dynamics.

Classic rational bubbles and representative bubbles: A mathematical model

Classic rational bubbles arise from the first trade by definition of the efficient market hypothesis, while representative bubbles can appear at any time due to psychological factors caused by exogenous shocks (shifts). Furthermore, classical rational bubbles arise from pure speculation given the failure of the transversality condition, while representative bubbles arise from the formation of distorted expectations whereby agents are unable to rationally understand a signal. In general, this type of bubble arises from mistakes and not from speculative incentives. There are other possible differences depending on the particular model considered, but in general the two above are always valid. The implication is that representative bubbles cause booms and busts at any time and their fluctuations can be steeper than rational ones. Giglio et al. (2016), who tested whether a classical rational bubble model can represent bubble episodes. This type of rational model identifies only those bubbles that can arise from an intertemporal inconsistency on the expected price equilibrium; in short: if the transversality condition on the sparkling component does not apply. Giglio et al. (2016) have shown that, as with classical rational models, bubble dynamics are not detected by looking for a failure of the cross-cutting condition, although advanced econometric tests such as (Phillips et al. 2015) have measured positive bubble dynamics in the same assets and in the same time intervals. From this gap comes the representative bubble model, a behavioral model. Starting from the work of Ferrara et al. (2017) we can identify the case of certainty and the case with rational expectations. These points are analyzed in subsection 3.1, 3.2 and 3.3.

The case of certainty

Define a difference equation of the form

$$p_t = ap_{t-1} - d_t \tag{1}$$

in which p_t is the price at date t and d_t is the forcing variable that can be thought of as the dividend stream at t. Moreover, the presence of a can be thought of as the gross required rate of return, which for now can be considered as constant. By n recursive substitutions and letting $n \to \infty$, Eq. (1) can be solved as:

$$p_t = \lim_{n\to\infty} \left(\frac{1}{a}\right)^n p_{t+n} + \frac{1}{a}\sum_{j=0}^{\infty}\left(\frac{1}{a}\right)^j d_{t+1+j} \tag{2}$$

For $a > 1$ and bounded sequences d_t, the transversality condition:

$$\lim_{n\to\infty}\left(a^{-n}p_{t+n}\right) = 0 \tag{3}$$

will hold. In this case, the fundamental solution f of Eq. (1) will be:

$$f_t \equiv p_t = \frac{1}{a}\sum_{j=0}^{\infty}\left(\frac{1}{a}\right)^j d_{t+1+j} \tag{4}$$

If the transversality condition does not hold, the general solution admits infinite solutions for any B_t, called the bubbly component, that satisfies $B_{t+n} = a_n B_t$. Therefore, $p_t = f_t + B_t$ is a solution of Eq. (1) defined as:

$$B_{t+1} - aB_t = 0$$

A nonzero value of B_t indicates that there is a positive bubbly component for which the price p_t is not consistent with the market fundamental f_t. In this case, the solution of Eq. (1) will become

$$p_t = f_t + \lim_{n\to\infty} B_{t+n}\left(\frac{1}{a}\right)^n \tag{5}$$

The bubble will thus explode depending on the gross return a since B_{t+n} can increase without bound. A bubble cannot be negative since it reflects market prices. It follows from the Walrasian free disposal for which a positive probability that future stock prices will be negative is inconsistent with free disposal of stocks. Thus, $B_t \geq 0$ for sure. In a model of certainty, if $a > 1$ and the sequences of dividends d_t are bounded, the solution is convergent. If in contrast $a < 1$, the general solution will be unstable and p will explode to $+\infty$.

With rational expectations

Suppose now that Eq. (1) is defined in terms of rational expectations:

$$\mathbb{E}_t p_{t+1} = ap_t - d_t \tag{6}$$

The rational expectations operator $\mathbb{E}$ is defined over current and past realizations of both p and d. For $a > 1$, the fundamental solution under rational expectations f^R is defined as:

$$f_t^R \equiv p_t = \frac{1}{a}\mathbb{E}_t\left[\sum_{j=0}^{\infty}\left(\frac{1}{a}\right)^j d_{t+1+j}\right] \tag{7}$$

Suppose now that the transversality condition does not hold and a rational bubbly component arises $B_t \geq 0$, which is a solution of the difference equation:

$$\mathbb{E}_t B_{t+1} - aB_t = 0 \tag{8}$$

For which $\mathbb{E}_t B_{t+1} = a^n B_t$ defines a rational bubble as following explosive conditional expectations. For a negative rational bubble, $\{\mathbb{E}_t B_{t+1}\}$ would be decreasing to $-\infty$ at a pace $a < 1$ and would drag p_t into negative territories, which is in contrast to the free-disposal condition. It is the case that $B_t \geq 0$ also in the rational expectations model. The solution of **Equaion (8)** satisfies the stochastic difference equation:

$$\mathbb{E}_t B_{t+1} - aB_t = X_{t+1} \tag{9}$$

In **Eq. (9)**, X_{t+1} is a random variable depicting an innovation that brings to the market a new piece of information, for which $\mathbb{E}_{t-n} X_{t+1} = 0 \forall_n \geq 0$. It follows from the reasoning that speculation is a "fair game" with respect to the information sequence. In particular, deviations of price changes from the values required to compensate investors for time and risk bearing have expected value equal to zero, conditional on past information. Following Diba and Grossman (1987), from **Eq. (9)** and the non negativity condition of B_t , then $X_{t+1} \geq -aB_t 0 \forall_t \geq 0$; that is, if $B_t = 0$, then $X_{t+1} = 0$ too. From **Eq. (9)** the bubble component is equal to the innovation brought about by X_{t+1}, whose expected value must be zero by definition of the efficient market hypothesis: If a bubble exists, then it must exist from its inception.

Representative bubbles

Representative bubbles are driven by market psychology; that is, they can occur whenever agents' expectations are formed in a biassed way. We think that the representativeness heuristic—which states that an event's "probability is judged by the degree to which it is representative of the major characteristics of the process or population from which it emerged" (Kahneman and Tversky, 1972)—is the source of this bias. We have two pivotal assumptions, the first one strictly connected with Kahneman and Tversky (1972) statement, i.e.:

Assumption 1: In the real world exists, in a discrete time with two state of world, a prior probability π_k with $k = \{g, r\}$ where g stands for the state of the world economy "growth" and the other one, r, stands for the dual i.e. "recession," for which one has $\pi_g \geq \pi_r$.

At any time, there is a signal $s \in \left[\bar{s}, \underline{s}\right]$ with $\underline{s} < \bar{s}$ about the next state space, with growth or recession accordingly. The signals are observed by everyone and are characterized in the following way: $Pr(\bar{s}|r) = \gamma$, $Pr(\bar{s}|g) = 1 - \beta$ for which $\gamma > \beta \geq 1/2$: A bad signal $\bar{s}$ reduces the probability of expecting a growth rate, and it is a very strong signal for a looming recession (and vice versa).

Assumption 1:

$$\lim_{n\to\infty}\left(\frac{1}{a}\right)^n \mathbb{E}_t^d p_{t+n} = 0$$

from which it follows that:

$$p_t = \frac{1}{a}\mathbb{E}_t^d\left[\sum_{j=0}^{\infty}\left(\frac{1}{a}\right)^j d_{t+1+j}\right] \tag{10}$$

By these last two assumptions we are modeling "Representative Bubbles" as "temporary" bubbles. In particular, it refers to representative bubbles whose cycle of expansion and contraction lasts for a period of time and therefore cannot be detected by classical models. In **Eq. (10)**, expectations are assumed to be *diagnostic*. In particular, agents are sensitive to positive news that affects fundamentals (shifts) causing overconfidence in the market, which in turn triggers both downside risk neglect and predictable positive errors given the extrapolative nature of these expectations (for further details, see Ferrara et al., 2018).

Conclusion

In this work we have shown that bubbles can burst at any time and without restrictions, generating several implications. It can be deduced how a financial bubble can start at any time due to the resulting shifts from extrapolative expectations and the neglect of extreme risks. As rational bubbles, the representative dynamics cannot be negative due to free transfer, in fact, representative bubbles can arise, burst at zero and then grow the same good again. Furthermore, a representative bubble does not depend on the failure of the transversality condition; therefore, it can be persistent and distorted. This work only considers homogeneous expectations among agents. There should be an improvement toward models with heterogeneity.

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