

Supporting diagrammatic reasoning for users with visual impairments with NoVAGraphS

Margherita Piroi^{1,4}, Daniele De Giorgi² and Carola Manolino^{3,4}

¹“I.I.S. Prever”, Pinerolo (TO);

²Department of Science, Technology and Innovation, University of Piemonte Orientale;

³Department of Social and Human Science, University of Valle d’Aosta; carola.manolino@unito.it

⁴Laboratory “S. Polin”, Department of Mathematics “G. Peano”, University of Torino; Italy

This paper presents a case study on how individuals with visual impairments interact with mathematical diagrams using NoVABOT, a dialogue-based system developed for the NoVAGraphS project. The study focuses on the experience of a blind participant navigating function diagrams represented as Euler-Venn sets. Through verbal interaction with NoVABOT, the participant explored key characteristics of the diagrams, demonstrating how the system can support the development of diagrammatic reasoning by guiding users through a process of inquiry and reflection. The findings illustrate how NoVABOT enables users with visual impairments to access and interpret graphical information, fostering deeper engagement with mathematical content. This case study highlights the potential of dialogue systems to bridge accessibility gaps in STEM education.

Keywords: Diagrams, functions, inclusion, visual impairments.

Introduction

In recent years, various assistive tools have been adopted by individuals with visual impairments, but these technologies have significant limitations in non-purely textual contexts, and they can restrict user autonomy (Bernareggi et al., 2016). Accessing structured graphical representations, such as tables, graphs and diagrams, presents particular challenges for at least three reasons. First, these images are designed to be perceived through the visual channel (Heller, 2002). Second, common non-visual representations, such as tactile drawings (e.g., Miragliotta et al., 2023), can be cumbersome and require expensive equipment, with overlapping tactile labels potentially reducing readability. Printed drawings lack interactivity, plastic sheets demand manual dexterity, and digital boards, while offering greater interactivity, still require fine motor skills and do not support permanent storage. Sonification approaches provide an alternative but face challenges, including the difficulty of transposing content into sound, the need for extensive training for non-spontaneous metaphoric content-sound association (Manolino et al., 2024), and cognitive demands when interpreting images through audio (Flowers, 2005). Third, creating effective non-visual representations of structured images requires an in-depth understanding of the domain and non-visual exploration techniques.

In the following section, we introduce a chatbot developed by a multidisciplinary team from different Departments (Computer Science, Mathematics, Physics, Social Education, and Economy) of the University of Turin, coordinated by the Research Laboratory for STEM Accessibility “S. Polin”, as our proposed voice-based tool for individuals with visual impairments (Oliverio et al., 2024). This chatbot allows users to explore the Euler-Venn representation of mathematical functions through dialogue-driven interactions, enabling them to ask questions and receive responses from the software. Here is the provisional link to the tested prototype: <https://delorean.di.unito.it/novagraph>.

This paper shares insights from testing the chatbot with a blind participant with no prior experience using the tool. The results prompted several important reflections, particularly regarding mental representations, which were found to differ significantly from those of sighted users.

NoVAGraphS project

NoVAGraphS (Non-Visual Access to Graphical Structures – <http://www.integrabile.unito.it/en/progetto-novagraphs/>) is an ongoing project devoted to the development of educational dialogue systems for users with visual impairments that specifically address the treatment of graphical structures. Scientific books frequently contain structured graphical information, such as tables and diagrams, which play a crucial role in conveying complex data and concepts. This project aims to establish standards through best practices and software solutions to enhance the interpretation and comprehension of structured graphical information, branching out into three main goals: (1) the study and the development of algorithms to convert graphical structures in dialogue scripts, which are prototypical dialogue fragments designed to communicate the information conveyed by the graphical structures; (2) the integration of these scripts in educational dialogue systems for users with visual impairments; (3) a comprehensive evaluation of these dialogue systems along different perspectives and disciplines. To fulfil the goals, the developer team designed a new dialogue system called NoVABOT. It is a web-based and accessible text-based dialogue system to which users with visual impairments can have access by using their own speech technologies, such as a screen reader and a keyboard. It is designed to understand and answer questions in Italian, in relation to three different topics: Euler-Venn function diagrams, electrical circuits, and finite state automata (https://delorean.di.unito.it/novagraph/tre_domini). These three topics were chosen as paradigmatic of the three scientific domains involved in the project: mathematics, physics and computer science. In particular, these topics are usually addressed precisely through diagrams: diagrams are both objects of study and tools or vehicles of knowledge. For relevance and conciseness, this contribution focuses on functions as a representative example. In this digital environment, NoVABOT engages in dialogues about functions on finite sets, represented through Euler-Venn diagrams. At this stage, several examples are available, with future plans to allow the software to “read” user-provided diagrams in input, e.g. through a photo. The user interface (see Figure 1) is designed with accessibility for users with visual impairments in mind. It is optimised for screen readers, featuring an input bar and a submit button directly beneath it. Responses to user queries appear below the button, and then a table displays the conversation history. This helps users to track and associate questions with answers. Webpage accessible navigation is simplified using HTML tags. The central feature of the interface will be an accessible and navigable SVG image, although this functionality is still under development. Currently, the image in focus is displayed on-screen but is not accessible (Figure 1).

NoVABOT is designed with a focus on inclusivity, featuring an interface that integrates both graphical and textual elements. This design choice aims to accommodate a diverse range of users with varying needs. For instance, the tool holds potential as a support for students with ADHD, benefiting from its visual aids and some colour-coded highlights of key elements of the function. Furthermore, the NoVABOT interface complies with the Web Content Accessibility Guidelines 2.1, ensuring comprehensive technical accessibility for users with visual impairments. The overall project design aims to enable them to access diagrams, fostering their development of diagrammatic thinking.

Question:

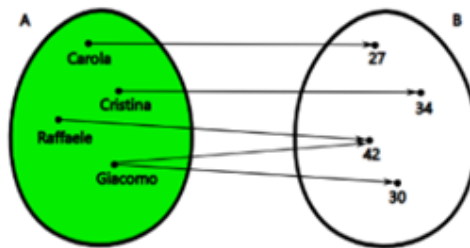
Domanda:

Chiedimi qualcosa rispetto a questa funzione... **Ask me about this function...**

Invia

Set A consists of four elements: Cristina, Raffaele, Giacomo, Carola

L'insieme A è composto da 4 elementi: Cristina, Raffaele, Giacomo, Carola.



Dialogue history
Storia del dialogo

Question	Answer
# Domanda	Risposta
4 Is a diagram a graph containing the representation of a function with the relationship between its domain and codomain? Il diagramma è un grafico contenente la rappresentazione di una funzione con la relazione tra il suo dominio e il suo codominio?	the element Carola is linked to element 27, the element Cristina is linked to element 34, the element Raffaele is linked to element 42, the element Giacomo is linked to element 30 L'elemento Carola è collegato all'elemento 27, l'elemento Cristina è collegato all'elemento 34, l'elemento Raffaele è collegato all'elemento 42, l'elemento Giacomo è collegato all'elemento 30
3 How many elements are there in set A? Quanti elementi ci sono nell'insieme A?	Set A consists of four elements: Cristina, Raffaele, Giacomo, Carola L'insieme A è composto da 4 elementi: Cristina, Raffaele, Giacomo, Carola.

Figure 1. NoVAGraphS interface while exploring an example [in bold authors' English translation of the texts on the screen, originally in Italian in grey]

Theoretical Framework

Charles Peirce suggests that much of mathematical thought involves observing or visualising results and patterns that emerge from manipulating various types of diagrams (Dörfler, 2005; Peirce, 1965). He defines a diagram as a sign “which is predominantly an icon of relations and is aided to be so by conventions. Indices are also more or less used” (Peirce, 1965, 4.418, 1903). Diagrams’ key feature is their iconic nature, as they primarily serve to represent relationships (Bakker & Hoffmann, 2005). For Peirce, diagrammatic reasoning consists of three stages.

- The first stage is to construct diagram(s) using a representation system. This representation is required to show the relationships that are important in a given mathematical object or problem, and it can be done by different means, such as drawn sketches or software.

- The second stage is to experiment with the diagram(s). This experimentation must follow the rules of the specific representational system, so possible actions on a diagram are constrained. The important feature of this stage, as Bakker and Hoffman (2005) point out, is that thinking in general terms is not enough. The ability to observe is essential to act on the diagram.
- The third stage is to observe the results of experimenting and reflect on them.

Considering the concept of diagrammatic reasoning, our broader research aim is to investigate how it manifests in blind mathematics students, or more generally, in individuals with visual impairments when solving mathematical problems. In this paper, we specifically examine how NoVABOT can facilitate the exploration and interaction with diagrams. Consequently, our research question is: How does diagrammatic reasoning manifest in blind students or solvers through the use of NoVABOT?

Methods

We conducted task-based interviews with four blind users, letting them explore NoVABOT. We have used the three stages proposed by Peirce to analyse the process of diagrammatic reasoning of users. Regarding the first stage, "constructing a diagram", in this case, it was we who initially produced a diagram. That is, we prepared and uploaded to NoVABOT six different examples of functions between finite sets represented (graphically) via Euler-Venn diagrams. Given the embryonic nature of this study, we decided to provide the solvers with ready-made diagrams. However, the overall aim of the research project is to also examine, in future studies, the construction process by the users. In NoVABOT there is of course not only the graphical representation. The software implicitly knows the relationships between the different elements. It is therefore able to answer questions posed by users, e.g. about the domain and codomain elements, or the relationships between these elements.

In the first phase of the interviews, we let the users experiment and explore the diagrams in the different examples by asking them to identify characteristics they considered important of the mathematical objects explored. Subsequently, we asked them to reflect on the observed characteristics, to recognise which functions were injective, surjective, bijective, or which were not functions at all. This phase was followed by a meta-reflection moment in which respondents were asked to give feedback on the software and whether and how it can support users with visual impairments in exploring relationships between mathematical objects, where diagrammatic representations are particularly useful in highlighting such relationships, but are inaccessible to them.

In this paper, we present an excerpt from the task-based semi-structured interview, conducted through an online remote calling, with Antonio, a physics graduate who lost his sight six years ago due to a degenerative condition. He regularly uses his personal computer with the aid of speech synthesis. Since our aim is "to portray 'what it is like' to be in a particular situation, to catch the close up reality and 'thick description' [...] of participants' lived experiences of [...] a situation" (Cohen et al., 2007, p. 182), to effectively capture the perspective of blind users, we chose an exploratory approach and utilised a case study design. This methodology parallels those used in previous research on expert blind solvers (e.g., Maffia et al., 2023; Miragliotta et al., 2023, Maffia et al., 2025). In particular the last part of the interview, comprising the interviewee's meta-reflection on his own exploration, turns out to be particularly informative in the case of Antonio, as he is an expert solver of mathematics,

and not blind from birth therefore able to provide interesting comparisons between different forms of mental representation making use of images and not.

To construct diagram(s)

As mentioned, we prepared and uploaded six different examples of functions between finite sets to NoVABOT. Following the work of Martins et al. (2023), who studied 10th-grade students' functional thinking through their understanding of graphical representations of functions, we selected examples that illustrate functions with varying properties. For instance, Example 1 represents a function that is both injective and surjective, and thus bijective; Example 2 depicts a surjective but not injective function. Example 5 presents a relation that is not a function, as one element of the domain has two different images. These examples were carefully chosen to enable users to explore regularities in the relationships between elements and to distinguish the different properties of the explored functions.

Antonio's interview through the Diagrammatic Reasoning stages

To experiment with the diagram

At the beginning of the interview, Antonio is asked to explore NoVAGraphS interface. He observes several examples and chooses to examine Example 5. In this example (visible in Figure 1), as in the others, the diagram is presented alongside an overall description of the situation, which is read via a screen reader. The description states: "In the figure, there are two circles. Each circle contains some elements. Oriented arrows start from elements in the first circle (A) and point to elements in the second circle (B)". This description provides a preliminary overview of the diagram's notable features, as if it was a "first glance" at the diagram. Then, there is the area where the user can write and pose questions to the chatbot. Here is the initial interaction between Antonio and the chatbot.

- Antonio: Is the diagram a graph containing a representation of a function with the relationship between its domain and its codomain?
- Chatbot: Set B consists of 4 elements: 27,42,34, 30.
- Antonio: In set A, how many elements are there?
- Chatbot: In the first circle (in the domain) there are four elements: Cristina, Raffaele, Giacomo, Carola.
- Antonio: How many arrows are there in the graph?
- Chatbot: Forgive me, I didn't understand. Could you try rephrasing the question using keywords such as set, connected to, associated with, or naming some element of the sets?
- Antonio: Describe how the elements of the two sets are connected, please.
- Chatbot: The sets in our example are two. The first set, set A, called the domain in this case, consists of 4 elements: Cristina, Raffaele, Giacomo, Carola. The second set, set B, known as the codomain, on the other hand, is composed of 4 elements: 27, 42, 34, 30.
- Antonio: Describe how the elements are connected.
- Chatbot: Element Carola is connected to element 27, element Cristina is connected to element 34, element Raffaele is connected to element 42, element Giacomo is connected to element 30, 42.
- Antonio: Giacomo to which element is related?
- Chatbot: The element Giacomo of set A is connected to elements 30 and 42 of set B.

At this point, Antonio states that the situation is clear to him. One of the interviewers asks him whether he has therefore understood what is depicted in the diagram, or whether he needs to ask more questions. Antonio replies as follows.

Antonio: So, I have one doubt, though. As it's described, I imagine on one side the set A with the four names – Carola, Cristina, Raffaele, Giacomo, if I'm not mistaken –; on the other side I imagine the four numbers 27, 34, 30 and 42, with Carola connected to 27, Raffaele I think to 34, Cristina I don't remember...to 30 maybe, and Giacomo to 30 and 42. And I picture arrows starting from the names and going like in a direction to... from the set of names, from set A to set B. That is, I picture it in that way. So, the arrows with the arrow pointing from A to B.

Then, he is asked if he can say something more about the representation, considering the information he has gathered.

Antonio: But then, if that is the case, I would say that... I would have said initially that it could be a representation on one side of the names and on the other side of the age of these people. But having already like a two-element link, it's not the age. Which is not a biunivocal relationship, because anyway, it has more elements of the domain...To one element of the domain correspond more elements of the codomain, in the case of Giacomo. Yes, a little bit of this.

In the rest of the interview, Antonio explores additional examples of injective and non-injective, surjective and non-surjective functions. For each example, he is asked to express the properties he notices. For reasons of space, further sections of this interaction are not included here. Instead, we focus on the concluding part of the interview, where Antonio is asked to provide his feedback on the software's functionality and usefulness for a blind person.

To observe the results of experimenting and reflect on them

In response to the interviewer's question, Antonio shares his comments.

Antonio: In my opinion, the interaction with a conversational chatbot – so precisely that one, it has a memory of what has been said – is adequate for this kind of exploration of mathematically complex graphs, because it allows you to ask for information little by little, to mentally reconstructing... Generally, let's say, to mentally construct “the overall”, what the whole diagram looks like. In my opinion, it's a good interaction system for the exploration of complex graphs. It's analytical, just like the exploration of the person who has a visual disability can be: analytical, rather than synthetic; later, you do the synthesis yourself, mentally. I'm doing that, though.

[...]

And as I said, for example, I knew before that there were two groups of two sets, connected with elements, and these elements connected with each other. In order to understand, to reconstruct how it was made, I asked at first “what are the elements in the set?”, so that I could understand what was in one set, what was in another set, and mentally represent it to myself. Then, the second step, I asked about the connections, so how the elements were connected, in the way to add this information to what I already had in my head.

So I was analytical, in the sense that I proceeded piece by piece: first, the sets, then the elements within the sets, then the connections between the elements, and then mentally I made the synthesis of this information to try to construct by/for myself, precisely, this mental representation. That's it.

Discussion and conclusions

Antonio's reflection serves as a valuable starting point for analysing how he explores the graph and investigating a blind user's diagrammatic reasoning, in dealing with NoVABOT. He begins by examining the various components of the diagram. Knowing that there are two sets, he first seeks to understand their composition. Subsequently, he investigates the relationships between the elements

of these two sets. Observing that an element of set A corresponds to multiple elements of set B, he asks a specific question to confirm this observation. Only at this stage is he able to piece together the overall relationship, recognising that one element in the domain is associated with two elements in the codomain. Although his terminology is not entirely precise, Antonio concludes that the example does not represent a function. An interesting aspect of this deduction is the role played by the example's context. Though not explicitly stated, Antonio infers that the elements in the second set represent ages, and the relationship between the elements in the two sets is "Person is X years old". The fact that "Giacomo is 30 and 42 years old" leads Antonio to deduce that this cannot be a function, as one person cannot have two different ages. It is notable how this insight, which might not be immediately apparent visually, shifts the focus from the "content" of the elements to the presence of a link between domain and codomain elements.

In conclusion, interaction with NoVABOT facilitates targeted exploration. The user can analyse various aspects of the diagram according to their choice, combining the gathered information to understand the fundamental relationships. The software does not offer a generic description but requires the user to engage in analysis by asking specific questions. This process aligns with the intentional activity advocated by Bakker and Hoffmann (2005) in the exploration phase of diagrammatic reasoning. The limitations observed in this excerpt stem from the fact that the dialogue system does not fully capture or maintain the immediacy of the graphical content provided by the Euler-Venn representation of functions. Although NoVABOT does not manage to preserve the immediate visual impact of the diagram, it does sustain the diagrammatic reasoning and, most importantly, ensures user autonomy. Autonomy is a crucial aspect in the context of higher education. As reported for example in Bernareggi et al. (2016) and Piroi et al. (2023), blind students traditionally rely on human intermediaries to access and work with complex diagrams, such as those in focus within the NoVAGraphS project. NoVABOT represents indeed an innovative digital tool for fostering blind users' autonomous exploration of graphical mathematical representations. The benefits of this opportunity for autonomous exploration were highlighted by Antonio and the other participants interviewed in this study.

Further studies are needed to investigate the software usability and inclusive educational potentialities. At this stage of research and software development, some elements remain unstable: determining the extent of knowledge and interaction competence the software should possess to remain responsive without replacing the user's responsibility for knowledge construction; identifying the most appropriate vocabulary for the chatbot – whether it should adopt a neutral, descriptive register (circles, arrows, dots) or a content-related one (domain, image, counter-image); and understanding what can be lost or removed from such representations in order for them to be equally useful but accessible. Addressing these questions may require returning to foundational issues in mathematical epistemology, particularly the conceptual poles around which such representations are structured.

Acknowledgment

This research has been partially supported by the NoVAGraphS project grant 2021.1930 from Fondazione CRT (Cassa di Risparmio di Torino) and by the Italian Institute of High Mathematics' National Group for Algebraic and Geometric Structures and their Applications (GNSAGA-INdAM).

References

- Bakker, A., & Hoffmann, M. H. G. (2005). Diagrammatic reasoning as the basis for developing concepts: A semiotic analysis of students' learning about statistical distribution. *Educational Studies in Mathematics*, 60(3), 333–358. <https://doi.org/10.1007/s10649-005-5536-8>
- Bernareggi, C., Gerino, A., & Mascetti, S. (2016). *Accessible Mathematics on Touchscreen Devices: New Opportunities for People with Visual Impairments*. [Unpublished contribution at Deims Conference 2016, retrieved from https://air.unimi.it/bitstream/2434/471939/2/p14_Cristian.pdf]
- Cohen, L., Manion, L., & Morrison, K. (2007). *Research Methods in Education*. Routledge.
- Dörfler, W. (2005). Diagrammatic Thinking. In M. H. G. Hoffmann, J. Lenhard, & F. Seeger (Eds.), *Activity and Sign* (pp. 57–66). Springer-Verlag. https://doi.org/10.1007/0-387-24270-8_6
- Flowers, J. H. (2005). Thirteen years of reflection on auditory graphing: promises, pitfalls, and potential new directions. In *Proceedings of ICAD05* (pp. 406–409). <https://digitalcommons.unl.edu/psychfacpub/430>
- Heller, M. A. (2002). Tactile picture perception in sighted and blind people. *Behavioural Brain Research*, 135(1–2), 65–68. [https://doi.org/10.1016/s0166-4328\(02\)00156-0](https://doi.org/10.1016/s0166-4328(02)00156-0)
- Maffia A., Manolino C., & Miragliotta E. (2023). Algebraic Structure Sense in a blind subject. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 307–314). University of Haifa, Israel: PME. <https://pme46.edu.haifa.ac.il/conference-proceedings>
- Maffia, A., Manolino, C., & Miragliotta, E. (2025). There is more to algebra than meets the eye: The case of blindness. *Educational Studies in Mathematics*. doi.org/10.1007/s10649-025-10394-0
- Manolino, C., Piroi, M., Armano, T., Brunetto, E., Ducci, M., Funghi, S., & Bernareggi, C. (2024). La sonificazione dei grafici di funzione con Audiofunctions 2.0. In D. Marocchi, M. Rinaudo, & M. Serio (Eds.), *Insegnamento e Apprendimento della Matematica e della Fisica nel Periodo post Pandemia. Atti del XI Convegno Nazionale DI.FI.MA. 2023* (pp. 322–320). Collane@unito.it. <https://www.collane.unito.it/oa/items/show/209>
- Martins, R., Viseu, F., & Rocha, H. (2023). Functional thinking: A study with 10th-grade students. *Education Sciences*, 13(4), 335. <https://doi.org/10.3390/educsci13040335>
- Miragliotta, E., Manolino, C., & Maffia, A. (2023). Figural component in geometrical reasoning: the case of a blind solver. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of CERME13* (pp. 4467–4474). Alfréd Rényi Institute of Mathematics and ERME. <https://hal.science/hal-04408161/document>
- Oliverio, M., Piroi, M., De Giorgi, D., Balestrucci, P., Manolino, C., Mazzei, A., Anselma, L., Bernareggi, C., Serio, M., Sabena, C., Armano, T., Coriasco, S., & Capietto, A. (2024). Novagraphs: Towards an accessible educational-oriented dialogue system. In D. Taibi, D. Schicchi, M. Temperini, C. Limongelli, & G. Casalino (Eds.), *Proceedings of AIXEDU2024* (Vol. 3879, pp. 1–12). CEUR-WS. https://ceur-ws.org/Vol-3879/AIXEDU2024_paper_34.pdf
- Peirce, C. S. (1965). *Collected Papers*. The Belknap Press of Harvard University Press.
- Piroi, M., Manolino, C., Armano, T., Taranto, E., & Capietto, A. (2023). Teacher professional development via a MOOC on assistive technology for visually imp students learning mathematics. *Journal of Mathematics Education*, 16(1), 59–78. <https://doi.org/10.26711/007577152790164>