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Climate Change Sustainability: From Bargaining to Cooperative Balanced Approach

Tiziana Ciano [1,†], Massimiliano Ferrara [1,2,*,†], Mariangela Gangemi [1,†] and Domenica Stefania Merenda [1,†] and Bruno Antonio Pansera [1,†]

[1] Department of Law, Economics and Human Sciences & Decisions_Lab, University "Mediterranea" of Reggio Calabria, via dell'Universitá, 25, I-89124 Reggio Calabria, Italy; tiziana.ciano@unirc.it (T.C.); mariangela.gangemi@unirc.it (M.G.); stefania.merenda@unirc.it (D.S.M.); bruno.pansera@unirc.it (B.A.P.)

[2] Department of Management and Technology, ICRIOS-The Invernizzi Centre for Research in Innovation, Organization, Strategy and Entrepreneurship Bocconi University, via Sarfatti, 25, I-20136 Milano, Italy

* Correspondence: massimiliano.ferrara@unirc.it

† These authors contributed equally to this work.

Abstract: This work aims to provide different perspectives on the relationships between cooperative game theory and the research field concerning climate change dynamics. New results are obtained in the framework of competitive bargaining solutions and related issues, moving from a cooperative approach to a competitive one. Furthermore, the dynamics of balanced and super-balanced games are exposed, with particular reference to coalitions. Some open problems are presented to aid future research in this area.

Keywords: balanced and super-balanced games; competitive bargaining sets; non-empty core; climate change; sustainability

JEL Classification: C70; C78; C60

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Citation: Ciano, T.; Ferrara, M.; Gangemi, M.; Merenda, D.S.; Pansera, B.A. Climate Change Sustainability: From Bargaining to Cooperative Balanced Approach. *Games* **2021**, *12*, 45. https://doi.org/10.3390/g12020045

Academic Editors: Giovanna Bimonte and Ulrich Berger

Received: 2 April 2021
Accepted: 18 May 2021
Published: 22 May 2021

1. Introduction

In recent years, the research attention given to issues relating to sustainability, green economy, and climate change has led to the drafting and sharing of common procedures to reduce the pollution threshold and avoid or delay environmental catastrophes (for example, the dissolution of glaciers and the consequent rise in temperatures). The United Nations quickly became the protector for the resolution of these problems, with the first framework agreement [1] being signed in Rio de Janeiro in 1992. The introduction of the Kyoto Protocol emphasized the need for cooperation in order to implement some of the main similarities dictated by the aforementioned protocol at a global level.

In particular, the Kyoto Protocol of 11 December 1997 [1], which came into force on 16 February 2005, established rules for the reduction in environmental pollution, aiming at containing global warming through the adoption of a system of flexible mechanisms for the acquisition of emission credits organized in three parts:

(a) Clean Development Mechanism: allows industrialized countries to carry out projects in developing countries that produce environmental benefits for the latter and advantages in terms of emission credits for the former;
(b) Joint Implementation: allows industrialized countries to carry out projects aiming at reducing greenhouse gas emissions in another country of the same group using accrued credits;
(c) Emissions Trading: allows the exchange of emission credits between industrialized countries involved in different projects in order to rebalance development disparities through compensatory redistribution.

A further agreement on environmental protection was signed in Paris at the climate conference held in 2015 [1]; its implementation was intended to come into effect in 2020. This agreement is a legally binding instrument for the "United Nations Framework Convention on Climate Change" (UNFCCC) and implies a commitment by all countries to limit their greenhouse gas emissions and improve their capacity to adapt to climate change, eliminating the inequality between developing countries and industrialized countries. This result is possible to achieve because the common principles valid for all countries that agree to comply with this agreement determine the orientation of private and state financial flows towards development with low greenhouse gas emissions.

An interesting study describing the first systematic approach to the study of sustainable economy and game theory was introduced by Dolinsky in [2]. In this paper, the author focuses his attention on the growing need for fossil fuels to fill the rampant increase in global energy requirements. Therefore, the author criticizes the current management of the economic system based mainly on the dualism of financial strategy and economic feasibility, attempting to highlight how economic research cannot ignore the reality of the market. Finally, he proposed a new model utilizing game theory to approach this problem. Game theory, as we will see later, can play a crucial role in the management of the world's energy resources, in the sustainable economy, and in reducing the production of pollution. In particular, Dolinsky sees the possibility of reducing energy consumption to a minimum in the propensity for cooperation and search for common strategies between all interested parties. The idea of applying game theory to solve this type of problem originated in the 1970s, when Rosenthal in [3] started to study the relationship between external economy and the classical concept of core. The core is one of the crucial notions in the study of balanced games. Demonstrating the existence of the core primarily means defining the relationship with the competitive equilibria for economic games, in which it is assumed there are no externalities. Finally, Rosenthal argued that, for economies where externalities have beneficial effects, the core does not necessarily determine the results.

A few years later, in 1997, Chander and Tulkens [4] continued the investigation of these issues by introducing a game model of global cooperation for the issue of climate change. The application of game theory to environmental pollution patterns and subsequent climate change has shown that, in accordance with the writings of Flam in [5], cooperation is beneficial for everyone, especially those who do not participate in operations to contain polluting emissions.

Assuming that agents will pollute less under a cooperative contract, cooperation among some parties will favor others, mostly free riders. In [4], the problem of international environmental externalities requires voluntary cooperation between countries within the international economy, so they represent the players within the game. The authors proved that inequality in terms of technological externalities—that is, the cost of the common benefits—is higher for the participants in the process. In particular, [4] examined the economic model of transboundary pollution through the joint abatement envisaged by the theoretical studies of international treaties. The result of sharing national abatement costs can be achieved through international financial transfers, viewed according to the concept of a classical solution from cooperative game theory—that is, the core of a game.

This active research has induced many researchers to verify the existence of such relationships, particularly after the introduction of the climate protocols mentioned above.

The following proposed model predicts that countries will minimize the total damage and abatement costs, known as the optimality property, and also that no member of the coalition can achieve lower total costs by employing different behaviors in terms of inputs or transfers as a response to countries external to the coalition. Meanwhile, in Helm [6] the environmental problem is faced from the point of view of a simple economy with multilateral externalities by analyzing a game in coalition. Helm demonstrated the hypothesis by which the core of the game is not empty and defining the core uses the characteristic function, which represents the payoff obtainable by all coalitions. The analysis must also consider the policies of the non-member agents of the coalition.

Moreover, countries are unwilling to cooperate with players that show high marginal damage or form a coalition with them. Into this perspective fits the blocking rule proposed by [1,4,7] regarding the choice of agents as the best response to conditional action. The substantial result is considered stable from a strategic point of view and is achieved through a joint negotiation aiming to reduce pollution.

Finally, in 2020 Cobo et al. [8] introduced a cooperative game model in the context of waste disposal in order to optimize two opposing players: on the one hand, the environmental ecosystem, and on the other the conflicting economic interests of rgw main stakeholders who interact within these environments.

A different approach to these issues is that of n-cooperative bargaining sets (c.b.s.), introduced by Aumann et al. in [9]. In [9], the context in which the c.b.s. approach is studied is represented by the exchange economy, which foresees the existence of agents who possess an initial endowment of "good" who decide whether to consume this endowment or whether it is more convenient to undertake mutually favorable exchanges. The author [9] considers a c.b.s. as a set of allocations, each supported by a coalition that shares its total endowment.

This paper is organized as follows: after a discussion related to competitive bargaining sets and uniform competitive solutions of cooperative games, we will move on to facing some exchange economic models and new applications for describing environmental problems, proposing lighted perspectives related to cooperative games. A model for the dynamics of the Kyoto Protocol is introduced. Finally, an analysis of balanced and super-balanced games is promoted, with a focus on the role of coalitions.

2. Competitive Bargaining Sets and Uniform Competitive Solutions of the Cooperative Games

Today, given the global situation and the difficulties that nations have in meeting the demands of a sustainable economy, there is need for collaboration regarding the search for common strategies to obtain the best results in terms of decreasing pollution and exploiting environmental resources. These models are suitably modified in terms of weight but rigidly anchored to the notions of a finite coalition aiming at the resolution of all issues concerning green economy. The motivation for this lies in the need to cooperate in order to minimize the environmental impact. Such assumptions, as Flåm [5] describes, lead to a competitive equilibrium in emissions markets.

Let $\mathcal{E}$ be an exchange economy and $\mathcal{C}$ be a finite collection of distinct subsets of N. Any redistribution of the total endowment to agents is called *allocation*. Formally, an allocation is an n-tuple $(x_1, x_2, \ldots, x_n)$ vector of the space $\mathbb{R}^m_+$, such that $\sum_{i=1}^n x_i = \zeta$. The set of all allocations is denoted by $\mathcal{A}$.

Definition 1. *A competitive bargaining set (shortly c.b.s.) of the economy $\mathcal{E}$ is any collection $\mathcal{B} = \{(x^C, C) : C \in \mathcal{C}\}$ satisfying the following conditions:*

(1) $\bigcup_{C \in \mathcal{C}} C = N$

(2) $x^C \in \mathcal{A}(\mathcal{C})$, *for every* $C \in \mathcal{C}$

(3) $u_i(x_i^C) = u_i(x_i^D)$, *for every* $i \in C \cap D$ *if* $C, D \in \mathcal{C}$

(4) *If* $y \in \mathcal{A}(S)$, *for some coalition* S *and* $u_i(y_i) \geq u_i(x_i^C)$ *for all* $C \in \mathcal{C}$ *and* $i \in S \cap C$, *then* $u_i(y_i) = u_i(x_i^C)$ *for all* $C \in \mathcal{C}$ *and* $i \in S \cap C$.

As follows implicitly from the above definition, a c.b.s. is basically a set of allocations, each supported by a coalition that shares its own total endowment and satisfying two stability properties. The internal stability is emphasized by (3) in Definition 1. An allocation in $\mathcal{B}$ cannot be used as an objection against another allocation in $\mathcal{B}$, since every consumer involved in both supporting coalitions obtains the same satisfaction by both allocations. The external stability is expressed by (4) in Definition 1. There are no coalitions outside $\mathcal{C}$ that can take advantages for their members sharing the total own endowment. The consumers attain their best satisfaction when they commit to coalitions in $\mathcal{C}$. Moreover, a c.b.s. has a remarkable optimality property. For each $C \in \mathcal{C}$, there is no allocation in $\mathcal{A}(C)$ that each individual in C prefers to x^C.

Definition 2. *The allocation $x \in \mathcal{A}(C)$ is Pareto optimal in $\mathcal{A}(C)$ if there is no other allocation $y \in \mathcal{A}(C)$, such that $u_i(y_i) \geq u_i(x_i)$ holds for each $i \in C$ and $u_i(y_i) > u_i(x_i)$ holds for at least one consumer $i \in C$.*

The next result immediately follows from the definition.

Proposition 1. *If $\mathcal{B} = \{(x^C, C) : C \in \mathcal{C}\}$ is a c.b.s., then each x^C is Pareto optimal in $\mathcal{A}(C)$.*

It is also important to note that the concept of c.b.s. is closely related to a stronger form of optimality as it occurs in the definition of the core.

Definition 3. *The core of the economy $\mathcal{E}$ is the set $C(\mathcal{E})$ of all allocations $x \in \mathcal{A}$ with the property that there are no S and $y \in \mathcal{A}(S)$, such that $u_i(y_i) > u_i(x_i)$ for all $i \in S$. Each allocation $x \in C(\mathcal{E})$ is called a core allocation.*

The concepts of c.b.s. derive from the concept of the uniform competitive solution of a cooperative game. It will be shown that a neo-classical exchange economy admits c.b.s. if and only if the associated cooperative game has a uniform competitive solution.

Following Stefanescu [10], let (N, V) be a cooperative game.

Definition 4. *A pair (u, C), where $C \in N$ and $u \in V(C) \cap V(N)$ is called a proposal of the game, denoted by pr_C.*

Let $\mathcal{S}$ now be a finite collection of proposals: $\mathcal{S} = \{(u^C, C) : C \in \mathcal{C}\}$, where $\mathcal{C} \subseteq 2^N \setminus \{\emptyset\}$.

Definition 5. *$\mathcal{S}$ is a uniform competitive solution (u.c.s.) of the game (N, V) if it satisfies following two conditions:*

1. *$u_i^C = u_i^D$ for every two coalitions $C, D \in C$ and $i \in C \cap D$*
2. *If $y_{S\cap C} > u^C_{S\cap C}$ for some proposal (y, S) and $C \in C$ then there exist $D \in C$ and $i \in S \cap D$, such that $u_i^D > y_i$*

A u.c.s. is a stable set of proposals. The stability can be explained as follows: internally, each of the two proposals within the u.c.s. are equivalent with respect to the players belonging to both supporting coalitions; externally, if a coalition not belonging to $\mathcal{C}$ has an objection against an internal proposal, then there exists at least one member of this coalition that may claim a counter objection invoking another internal proposal. Moreover, any u.c.s. can be characterized by some Pareto optimality property.

Proposition 2 ([11])**.** *If $\mathcal{S}$ is a u.c.s., u_C^C is the Pareto optimum of $pr_C(V(C) \cap V(N))$.*

Note that (1) in Definition 1 in Definition 4 uniquely defines an n-vector $u \in \mathbb{R}^n$ by $u_i = u_i^C$ for every $C \in \mathcal{C}$, such that $i \in C$. This vector was called in Stefanescu [10] the ideal payoff vector associated with $\mathcal{S}$. Moreover, it was shown that a u.c.s. can be equally defined as a pair $(u \in \mathcal{C})$, $u \in \mathbb{R}, C \subseteq 2^N \setminus \emptyset$, satisfying the following conditions:

$$\bigcup_{C \in \mathcal{C}} C = N \tag{1}$$

$$u_C \in pr_C V(C) \cap V(N) \text{ for all } C \in \mathcal{C}. \tag{2}$$

$$\text{There does not exist any proposal } (y, S) \text{ such that } y_s > u_s. \tag{3}$$

In fact, if $\mathcal{S}$ is a u.c.s., then the pair $(u, \mathcal{C})$, where u is the associated ideal payoff vector, satisfies the conditions in the above. Consequently, if a pair $(u, \mathcal{C})$ satisfies these conditions, then a u.c.s. $\mathcal{S}$ may be defined taking u^C as any vector in $V(C) \cap V(N)$ for which $u^C_C = u_C$ for all $C \in \mathcal{C}$. Moreover, in this case u is the ideal payoff vector associated with $\mathcal{S}$. Now let us consider an exchange economy $\mathcal{E}$ and let (N, V) be the associated cooperative game.

Theorem 1. *Let $\mathcal{B} = \{(x^C, C) : C \in \mathcal{C}\}$ be a c.b.s. of the economy $\mathcal{E}$ if and only if $\mathcal{S} = \{(u^C, C) : C \in \mathcal{C}\}$ is a u.c.s. of the associated game (N, V), where $u^C_i = u_i(x^C_i)$ for every $C \in \mathcal{C}$ and all $i \in N$.*

Proof. Assume first that $\mathcal{B}$ is a c.b.s. and define $\mathcal{S}$ as in the theorem. Obviously, $u^C \in V(N)$, since x^C is an allocation, and $u^C \in V(C)$, since $x^C \in \mathcal{A}(C)$. Since (1) in Definition 5 is trivially implied by (3) in Definition 1, let us verify (2) in Definition 5. Let (V, S) be a proposal of the game (N, V) and suppose that $V_{s \cap c} > u^C_{S \cap C}$ for some $C \in \mathcal{C}$. There exists $y \in \mathcal{A}(S)$, such that $V_i \leq u_i(y_i)$ for all $i \in S$. Hence, $u_i(y_i) \geq u_i(x^C_i)$ for all $i \in S \cap C$ and $u_i(y_i) > u_i(x^C_i)$ for at least one $i \in S \cap C$. Then, it should follows by (4) in Definition 1 that $u^D_\kappa = u_\kappa(x^D_\kappa) > u_\kappa(y_\kappa) \geq V_\kappa$ for some $D \in \mathcal{C}$ and $\kappa \in S \cap D$, so that (2) in the Definition 5 is satisfied. For the converse implication, let us assume that $\mathcal{S}$ is a u.c.s. of the game (N, V). By the definition of the associated game, for each $C \in \mathcal{C}$ there exists $x^C \in \mathcal{A}(C)$, such that $u^C_i \leq u_i(x^C_i)$ for all $i \in C$. This shows that the inequality never holds. To the contrary, if the above relations are satisfied such that the strict inequality holds for at least one C and one $i \in C$, then one obtains $v \in V(C) \cap V(N)$, taking $v_j = u_j(x^C_j)$ for all $j \in N$. A contradiction results, since u^C_C is the rgw Pareto optimum in $pr_C(V(C) \cap V(N))$. Now, show that $\mathcal{B} = \{(x^C, C) : C \in \mathcal{C}\}$ is a c.b.s. of the economy $\mathcal{E}$. Obviously, Equations (1) and (2) in Definition 1 are satisfied and (3) in Definition 1 immediately follows from the above and from (1) in Definition 5. To verify (4) in Definition 1, assume that there exist S and $y \in \mathcal{A}(S)$, such that $u_i(y_i) \geq u_i(x^C_i)$ for some $C \in \mathcal{C}$ and for all $i \in S \cap C$ and $u_i(y_i) > u_i(x^C_i)$ for at least one $S \in S \cap C$. Obviously, $v \in \mathbb{R}^n$, where $v_i = u_i(y_i), i \in N$ belongs to $V(S) \cap V(N)$, and $v_S > u_S$, where u is the ideal payoff vector associated with $\mathcal{S}$. Hence, a contradiction follows by Definition 3. □

3. Exchange Economic Model and Climate Dynamics Scenarios: An Existence Theorem

This approach concerns the neo-classical exchange economies used in to our analysis. Each country has to choose whether to preserve their environment (with sustainable politics) or lose their *environment* by encouraging industrial production with emissions (CO_2 and other polluting agents). In this direction, every country agent has at its disposal an initial endowment of this input. We can consider a model very close to the neo-classical mechanism design. The set of agents N represents the set of countries. We suppose that there are m different "environment issues" exchanged. Each agent i has a non-zero initial endowment $\zeta_i \in \mathbb{R}^m_+$ (i.e., $\zeta_i \geq 0$) and their preference relation or $\mathbb{R}^m$, which is represented by a utility function $u_i : \mathbb{R}^m \to \mathbb{R}$.

Without a loss of generality, it can be assumed that u_i are non-negative values. Let us denote by $\zeta = \sum_{i \in N} \zeta_i$ which is assumed to be strictly positive ($\zeta >> 0$). In a compact form, an exchange economy $\mathcal{E}$ will be represented as:

$$\mathcal{E} = (N, (\zeta_i, u_i)_{i \in N}).$$

Any redistribution of the total endowment to agents is called lenvironment allocation.

Formally, an allocation is an n-tuple $(x^e_1, x^e_2, \cdots, x^e_n)$ of the vector of the environment issue space $\mathbb{R}^m_+$, such that $\sum_{i=1}^n x^e_i = \zeta$. The set of all allocations will be denoted by $\mathcal{A}$.

For any coalition $C \subset N$, we will use the symbol $\mathcal{A}(C)$ for the set of all allocations with the property that the members of C share all their own endowments—i.e.,

$$\mathcal{A}(C) = \left\{ x^e \in \mathcal{A} : \sum_{i \in C} x_i^e = \sum_{i \in C} \zeta_i \right\}.$$

As we have defined an n-person game, it can be viewed as an exchange economy $\mathcal{E}$. With any $\mathcal{E}$, a cooperative game, (N, V), can be associated, in a similar way,

$$V(C) = \{u \in \mathbb{R}_+^m : \text{ then exists } x^e \in \mathcal{A}(C) \text{ such that } u_i \leq u_i(x_i^e), \forall i \in C\}.$$

The socio-economic structure related to countries' behaviors concerning the climate politics dynamics could be represented by an exchange economy in a classical formalization. The concepts of u.c.s. and c.b.s. reflect in a natural way some mechanism designs concerning the political process related to all decisions not normally made only in official meetings. The sequel conditions that are imposed seem to be quite natural. The proofs will be arranged based on the existence theorem of the u.c.s. introduced by Stefanescu [10] and the Theorem 1. The agents' (countries') preferences are assumed to be represented by continuous utility functions that are also monotonic.

Theorem 2. *Every exchange economic $\mathcal{E}$ whose agents' preferences are represented by continuous monotonic utility functions admits a competitive bargaining set.*

The existence of a uniform competitive solution for cooperative games has been shown for transferable utility games by Stefanescu [10] and for non-transferable utility games by the some author in [10]. A more general existence theorem including both mentioned results was proved by Stefanescu [10]. Now, we will present a generalization adopted to this new context of analysis.

Theorem 3. *Let (N, V) be a cooperative game satisfying the following conditions:*

1. *$V(S)$ is a non-empty subset of $\mathbb{R}_+^n$ and $pr_s V(S)$ is compact in $\mathbb{R}^{|S|}$ for every $S \neq \emptyset$.*
2. *If $u \in V(S)$ and $v \in \mathbb{R}_+^n$, $v_s \leq u_s$, then $v \in V(S)$.*
3. *$pr_C V(C) \subseteq pr_C V(D)$ whenever $C \subset D$*
4. *If $u \in V(S)$ and $0 \leq a \leq u_i$, for some $i \in S$ there exists $u' \in V(S)$, such that $u'_i = a$ on $u'_j > u_j$. For all $j \in S \setminus \{i\}$, the game admits a u.c.s.*

Proof. Consider $u_i(0) = 0 \; \forall i \in N$. Let (N, V) be a cooperative game associated with an exchange economy $\mathcal{E}$. We will show that condition (2) is satisfied. Then, the conclusion follows on from Theorem 1.

Since each u_i is monotonic and $u_i = 0$, we know that $V(S) \neq \emptyset$. Moreover, u_i is bounded on the compact $[0, \zeta]$, so $pr_S V(S)$ should also be bounded. To prove that it is closeness, let us assume that the sequence $(u_S^t) \subset pr_S V(S)$ converges in $\mathbb{R}^{|S|}$.

Then, there exists a sequence (x^t) in the set of all allocations $\mathcal{A}(S)$, such that $u_i^t \leq u_i(x_i^t), \forall i \in S$. Since $\mathcal{A}$ is a compact, there exists a subsequence of (x^t) which converges to an allocation x. Obviously, $x \in \mathcal{A}(S)$. The continuity of u_i implies that $u_i \leq u_i(x_i)$ if $i \in S$. Therefore, $u \in V(S)$ and the condition (1) is proven.

The comprehensiveness of the game is (N, V). To prove (3), let us suppose that $u_i \leq u_i(x_i)$, $\forall i \in C$, and from some allocation $x \in \mathcal{A}(C)$. Put $y_i = x_i$ if and only if $i \in C$ and $y_i = \zeta_i$ otherwise. Obviously, $y \in \mathcal{A}(D)$. If $v \in \mathbb{R}^n$ is defined by $v_c = u_c$ and $v_i = u_i(y_i)$ if $i \notin C$, then it is easy to see that $v \in V(D)$. Hence, $u_c \in pr_C V(D)$, so that $pr_C V(C) \subseteq pr_D V(D)$. Finally, show that (4) is also satisfied. Pick on $u \in V(S)$ and $0 \leq a \leq u_i$ for some $i \in S$. There exists $x \in \mathcal{A}(S)$, such that $u_i \leq u_i(x_i)$ for all $j \in S$. Since u_i is continuous and $u_i(0) = 0$ and $0 \leq a \leq u_i(x_i)$, there exists $\lambda \in (0, 1)$, such that $u_i(\lambda x_i) = a$. Put $x'_i = \lambda x_i$, $x'_j = x_j + \frac{1-\lambda}{|S|-1} x_i$ if and only if $j \in S \setminus \{i\}$ and $x'_j = \zeta_i$ otherwise.

Obviously, $x' \in \mathcal{A}(S)$ and by the monotonicity of the utility functions it also results that $u_j(x'_j) > u_j(x_j)$ if $j \in S$, $j \neq i$. Then, (4) is proven for u' defined by $u'_j = u_j(x'_j)$, $j \in N$. □

4. Uniform Competitive Solutions and Cooperative Games. A Model for the Kyoto Protocol Dynamics: A New Perspective

In this paragraph, we introduce an idea related to countries' behaviors concerning all the dynamics that were found around the agreements towards the closing act related to climate constraints. In this context, we consider the political dynamics the players could start following a cooperative approach, then switching in a uniform competitive one.

Definition 6 ([10])**.** *A pair (u, C), pr_C in the sequel, where $C \subset N$ and $u \in V(C)$ is called a proposal of the game.*

Let $S = \{(u^C, C) : C \in \mathcal{C}\}$ be a finite collection of proposals.

Definition 7. *Let S be a finite collection of proposals. We say that S is a complete uniform competitive solution (shortly (c.u.c.s.)) of the game (N, V) if it satisfies the following properties:*

1.

$$\bigcup_{c \in \mathcal{C}} C = N \tag{4}$$

2. *For all two coalitions $A, B \in \mathcal{C}$, such that $i \in A \cap B$, it follows that $u_i^A = u_i^B$;*
3. *If $y_{S \cap A} > u_{S \cap A}$ for some proposal (y, S) and $A \in \mathcal{C}$, then there exist $B \in \mathcal{C}$ and $i \in S \cap B$, such that $u_i^B > y_i$.*

If the property 4 in Definition 7 does not hold, then S is called the uniform competitive solution (u.c.s.) (see Definition 5).

A c.u.c.s. S is a substantially a stable set of proposals. The stability is explained as follows: internally, there are two proposals within the c.u.c.s., one equivalent with respect to the players belonging to both supporting coalitions. Externally, if a coalition does not belong to $\mathcal{C}$, it has an objection against an internal proposal, then there exists at least one member of this coalition that may claim a counter objection to another internal proposal.

The Equation (4) and condition 1 of Definition 7 define a n-vector $u \in \mathbb{R}^n$, denoted by $w_i = u_i^C$, for every $C \in \mathcal{C}$, such that $i \in C$. This vector is called by Stefanescu [10] the lideal payoff vector, associated with the coalition $S \subset N$.

We can observe that if S is a u.c.s., then $w_C = u_C^C$ for every $C \in \mathcal{C}$.

Proposition 3 ([10])**.** *Let S be a c.u.c.s. Then, for all $C \in \mathcal{C}$, the ideal payoff vector w satisfies:*

$$w_C \in pr_C(V(C)),$$

for every $C \in \mathcal{C}$.

Remark 1. *Starting from the Kyoto Protocol and successive Paris agreements concerning the climate targets, a question arises: the history clearly says that Canada (out since 2019), the USA (which did not participate in the last meeting), and China and India (that accepted an agreement with targets for all countries which requires sacrifices in the energy field but with high social impacts) together have played a disruptive role in the applications of the climate agreements. The previous assumptions suggest that their behaviors might be formalized by S in the last Proposition 3.*

5. Balanced Games and Related Developments: The Coalitions' Role into Political Process

Cooperative game theory has been frequently used in modeling various socio-economic scenarios. As is well known, economic systems have been treated as cooperative games, with the core of an economy commonly used as the main concept of a solution [12]. Some specific economic scenarios have been modeled within a cooperative/competitive framework. In this paper, we tried to model some effective or potential countries' behaviors into the mechanism design processes related to political decisions on occasions of climate change official meetings (otherwise not formal during the preliminary stages). It is well known that the concept of coalitions plays a central role in the frame of cooperative approaches, in particular concerning the *TU* and *NTU games* (Transferable Utility games and Non-Transferable Utility games).We are interested in selecting in our analysis some useful well-known results from the related literature in order to create a scientific platform to connect balanced and super-balanced games [13] with our thinking on the potential dynamics produced by decision-makers in order to obtain close agreements in terms of emissions (human-induced emissions of gases), impacts on ecosystems, displacement and migration, security, society, human settlement, energy, and transport.

The class of cooperative games can be further enriched if other assumptions are defined for the formation of possible coalitions. In this context, lbalanced games play an important role.

Definition 8. *Let $(\alpha_s)_{s\in S}$, $\alpha_s \in [0,1]$, be a set of weights for each $S \subseteq N$. Then, $(\alpha_s)_{s\in S}$ is called a balancing set of weights if for every $i \in N$, $\sum_{i\in N, S\subseteq N} \alpha_S = 1$.*

In other words, a family of weights is balanced if for each element of the set $S \subset N$ and for each coalition containing i the sum of the corresponding weights is equal to one. The meaning of these weights can vary depending on the context; many authors consider weights as the portion of time the player spends in the coalition to which they belong. In fact, we can consider a coalition S active for the time of α_S, and it is able to determine a payoff equal to $\alpha_S V(S)$ if all its players are active for a time interval equal to α_S. Balanced weights allow us to define the concept of a balanced family, which is necessary to introduce balanced games.

We denote by $\mathcal{C} = \{S : S \subset N\}$ the set of all coalitions of N and by $\mathcal{C}_i = \{S \in \mathcal{C} : i \in S\}$ the set of coalitions containing the member $i \in N$.

Definition 9 ([13])**.** *Let $\mathcal{F}$ be a family of sets. $\mathcal{F}$ is balanced if there exists a balanced set of weights associated with it.*

Definition 10 ([13])**.** *A game (N, V) is a balanced game if for every balanced family $\mathcal{F} \subset 2^N$ and for every associated system of weights $(\alpha_S)_{S\in\mathcal{F}}$ the inequality*

$$\sum_{S\in\mathcal{F}} \alpha_s V(S) \leq V(N) \tag{5}$$

holds.

In the set of balanced-type games, Ferrara in [13–18] introduced a new class of balanced games called *super-balanced games* through the following:

Definition 11 ([13])**.** *Let $\mathcal{F}$ be a family of sets. $\mathcal{F}$ is called super-balanced if there exists a system of positive weights $(\alpha_S)_{S\in\mathcal{F}}$, such that*

$$\sum_{S\in\mathcal{F}, i\in S} \alpha_S \geq 1, \quad \forall i \in N \tag{6}$$

Definition 12 ([13])**.** *A game (N, V) is super-balanced if for every super-balanced family $\mathcal{F} \subset 2^N$, and for every associated system of weights $(\alpha_S)_{S\in\mathcal{F}}$ the following*

$$\sum_{S\in\mathcal{F}} \alpha_S V(S) \geq V(N) \tag{7}$$

holds.

Finally, we consider, for completeness, the introduction of the game class introduced by Tijs and Lipperts in [19].

Definition 13 ([19])**.** *A game (N, V) is semi-balanced if, for all non-empty subset $A \subset N$, the following*

$$V(A) + \sum_{i \in A} V(N \setminus \{i\}) \leq |A| V(N)$$

holds.

Theorem 4 ([19])**.** *If (N, V) is a balanced game, then it is semi-balanced.*

Definition 14 ([19])**.** *A game (N, V) is totally balanced if every sub-game of a game is balanced.*

Theorem 5 ([19])**.** *A game (N, V) is semi-balanced if and only if $V \in H^N$.*

As specified by Rosenthal, the existence of the core does not necessarily entail an increase in benefits for economic models that provide for externalities. The models presented in this section describe economies with externalities from the point of view of cooperative games. In particular, these models are defined through the application of the concept of the balanced game. This assumption arises from the need for some axioms of the Kyoto protocol. To characterize these models, it is necessary to verify the existence of the core and the need for it to be non-empty. In the context of cooperative games, the core is the most frequently used concept of solution. The core defines the set of all value assignments of the grand coalition in order to prevent the formation of any other isolated coalition.

There have been many authors in the last century who have dealt with characterizing cooperative games in terms of core. One of the major achievements in this area is that of Gurk [20] in 1959 and that of Bondareva [21] and Shapley [22].

Theorem 6 ([21,22])**.** *A (N, V) game is balanced if and only if it has a non-empty core.*

The interest of the scientific community in those cooperative games that are defined by a characteristic function that generates non-empty cores is even more evident if we consider the geometric meaning of these assumptions. In fact, those games with a non-empty core seem to form a polyhedral cone. To confirm this thesis, it is appropriate to report a significant result of Spinetto [23], who presents a geometric characterization of these games.

Theorem 7 ([23])**.** *The cone of non-negative games with a non-empty core is generated by simple games with players who have a veto power.*

This brief premise is necessary, since Flam's main result [5] and Helm's main result [6] concern the characterization of the Theorem 6. In [6], evidence is studied regarding n-person games and problems concerning environmental pollution. The model introduced in [6] defines such classes of games in the simple economy with multilateral externalities. Therefore, within the duopoly of sustainable economy–environmental pollution, those games that are characterized by the property of having a non-empty core play a significant role. The first model of balanced games adapted to the need for a sustainable economy was introduced by Helm [6]. He considered a simple economy with multilateral externalities and assumed that from the amount of input $e_i \geq 0$, a unique non-negative private consumption good $x_i \geq 0$ can be produced.

To describe the technology, the author uses an increasing, differentiable, and concave production function $x_i = f_i(e_i)$. Quasilinear utility functions were used to express the preferences of each agent; this class of functions can be described by $u_i(x_i, e) = x_i - d_i(e)$. Agents outside of a blocking coalition choose their individual best reply to the action of all other agents. To characterize this game, Helm, in [6], referred to the so-called γ-*characteristic function*.

In [4], the authors defined the γ-*worth of coalition* $S \subseteq N$ by considering a non-cooperative game against S. In [5], the author presents an important interpretation in terms of the Nash equilibrium.

Specifically, in that game,

1. S acts as one coordinated player with the objective $\pi_i(x_i, e_S + e_{-S})$ and constraints $\sum_{i\in S} x_i \leq \sum_{i\in S} u_i(e_i, e_S + e_{-S}), \sum_{i\in S} e_i = e_S$;
2. each outsider $i \in I \setminus S$ plays with similar objective $\pi_i(x_i, e_i + e_{-i})$ and constraint $x_i \leq f_i(e_i, e_i + e_{-i})$

Definition 15 ([4])**.** *Using the previous hypotheses, we define the* γ*-characteristic function as follows:*

$$V^\gamma(S) = \max_{\{(x_i, e_i)\}_{i\in S}} \sum_{i\in S} u_i(x_i, e_i).$$

Definition 16 ([5])**.** *The* γ*-worth of coalition* S, $V^\gamma(S)$, *is the Nash equilibrium payoff it obtains in the described game against those players that are outside* S.

The introduction of the γ-characteristic function allows us to redefine the concepts of the core as follows.

Definition 17. *A payoff vector* $(\lambda_i)_{i\in N}$ *lies in the* γ-core *if:*

$$\sum_{i\in N} \lambda_i = V(N)$$

and for all $S \subset N$

$$V^\gamma(S) \leq \sum_{i\in S} \lambda_i.$$

On the basis of the foregoing considerations, it is clear that the fact of considering games with a non-empty core implies the impossibility of defining a model of games that favors the creation of coalitions that produce a total payoff greater than what the grand coalition can obtain.

The optimal input vector for coalition S is defined in [6] by $(e_i^S)_{i\in S}$; the reply of the coalition external agents is denoted by $(\bar{e}_i^S)_{i\in S}$. The sum $e^P = \sum_{i\in S} e_i^S + \sum_{i\in N\setminus S} \bar{e}_i^S$ defines the total emissions in a *partial agreement*. The pair (N, V^γ) is called a *coalitional game with multilateral externalities*.

Lemma 1 ([6])**.** *For a game* (N, V^γ), *and a set of balanced weights* $(\alpha_S)_{S\in\mathcal{C}}$ *the following*

$$\sum_{S\in\mathcal{C}\setminus\mathcal{C}_i} \alpha_S \sum_{j\in S} e_j^S \leq \sum_{S\in\mathcal{C}_i} \alpha_S \sum_{j\in N\setminus S} \bar{e}_j^S$$

holds.

Finally, this model, introduced by Helm, is characterized in terms of a non-empty core through the following:

Proposition 4 ([6])**.** *The game* (N, V^γ) *is balanced; therefore, it has a non-empty* γ*-core.*

Inspired by [6], in [5], the author presents a game model generated starting with a finite set of economic agents, $N = \{1, 2, \ldots, n\}$. The members of this group have a three-fold role as consumer, producer, and polluter. Using the same hypothesis introduced by Chander and Tulkens [4] and Helm [6], the author defined two one-dimensional ordered spaces, $(X, \leq)$ and $(E, \leq)$, the set of consumption x_i and emission e_i, respectively. Therefore, for each $i \in N$, Helm introduced two concave functions $f_i : E \times E \to X$ and $\lambda_i : X \times E \to \mathbb{R} \bigcup \{-\infty\}$: the first describes the efficient frontier of the transformation of its production set and the second represents the utility function.

In [5], Flåm denoted, for every $S \subset N$, the exogenous family of vectors of E by $e_{-S}^{outside}$. In order to prove the main theorem in [5], the author introduced the following assumptions:

1. The attitude of any coalition S, if it were to arise, would be that of one who opposes a total external emission prescribed and foreseeable emission from strangers $-S$ ($e_{-S}^{outside}$).
2. For any coalition $S \subseteq N$, we denote by $e_S = \sum_{i \in S} e_i$ any emission from S. For any balanced set α_S, $S \subseteq N$, and agent i, the following

$$\sum_{S \in \mathcal{C} \setminus \mathcal{C}_i} \alpha_S e_S \leq \sum_{S \in \mathcal{C}_i} \alpha_S e_{-S}^{outside}$$

supposedly holds.

Remark 2. *What connections are among u.c.s. and balanced (super-balanced) games? Uniform competitive solutions (u.c.s.) are basically stable sets of proposals involving several coalitions which are not necessarily disjointed. In the general framework of NTU games, the uniform competitive solutions were defined by Stefanescu [24,25]. The general existence results cover most situations formalized in the framework of cooperative game theory, including those for which the coalitional function is allowed to have an empty value. This approach concerns situations where the coalition configurations are balanced. In the related literature, it is known that if the coalitional function has a non-empty value, the game admits a balanced u.c.s. To each u.c.s., one associates an ideal payoff vector representing the utilities that the coalitions promise to the players. If the game is balanced, then the core and the strong core consist of the ideal payoff vectors associated with all balanced u.c.s.*

Open problem: It could be of interest to extend this result for the super-balanced case.

All these aspects, in our opinion, show the existence of a strict relation, with the models presented in this paper able to describe climate change agreement dynamics. This is the first step for further research in this interesting direction.

6. Conclusions

The main aim of this paper was to investigate the relationships that clearly exist between cooperative game models and some climate change issues. Climate change is a long-term change in the average weather patterns that have come to define the Earth's local, regional, and global climates. These changes have a broad range of observed effects that are synonymous with the term. The changes observed in the Earth's climate since the early 20th century are primarily driven by human activities, particularly the burning of fossil fuels, which increases the heat-trapping green-house gas levels in the Earth's atmosphere, raising the Earth's average surface temperature. These human-produced temperature increases are commonly referred to as global warming. Natural processes can also contribute to climate change, including internal variability. A great number of these phenomena are connected to or a direct consequence of some politics or governments policies. By analyzing the political dynamics concerning the Kyoto Protocol and the Paris Agreement, we defined some dynamics and found scientific relations—in particular, with some generalized exchange economy models—using the concepts of competitive bargaining sets and uniform competitive solutions for the associated cooperative games. This fascinating research direction enabled us to see some similarities among certain endogenous dynamics in the technical and political decision-making processes among the countries (players) at institutional official meetings or when creating ad horas coalitions for climate agreements. The proposed models concerning countries' behaviors related to climate dynamics, represented by an exchange economy (in a classical formalization), reflect some design mechanisms concerning the political process related to all decisions not normally made only in official discussions. In the last part of this study, another perspective was analyzed in terms of two central results related to the cooperative game approach: balanced and super-balanced games. Super-balanced games could be analyzed in terms of the effective role played by certain countries (Canada, USA, China, and India) which have been playing a strategic role worldwide in the revision of the original agreements made in Kyoto since 1997. These developments will be discussed in future research.

Author Contributions: Conceptualization, M.F.; methodology, T.C., M.G., D.S.M.; software, B.A.P.; validation, M.F.; formal analysis, B.A.P.; investigation, B.A.P.; resources, M.F.; data curation, M.F.; writing-original draft preparation, B.A.P.; writing-review and editing, B.A.P.; supervision, M.F.; project administration, B.A.P.; funding acquisition, M.F. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to express their gratitude to the anonymous referees for several helpful comments and suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

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