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Editorial

Preface to the Special Issue on "Differential Games and Its Applications"

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The study of differential games has practical applications in the analysis and resolution of conflict issues by the use of differential equations. The solution to this issue may be found somewhere between traditional game theory, which involves a number of players, and controlled dynamic systems, in which differential equations that describe the game are controlled by the participants. In point of fact, such games are among the most common, difficult, and essential optimization challenges that mobile agents are now confronting.

In this scenario, the games of pursuit and escape play an important part in the overall strategy, with one or more pursuers and one or more evaders participating as participants. The goals of the two groups of players are obviously in direct opposition to one another: the first group wants to capture the second group, while the second group wants to avoid being taken by the first group.

The strategies that are defined for these categories of games give the impression of being difficult to implement because, in addition to the resolution of the mathematical problem, there is also the matter of the intricate sensorimotor coordination that the pursuer needs to have in relation to the hostile behavior of the evader and the physical environment in which the pursuit takes place. These games have a wide range of potential applications, ranging from simple traffic control in the rush hour of a large city to military strategy, such as the missile guidance systems formulated by Rufus Isaacs, to surgery and management.

Additionally, there are many different types of hunting escape games, such as dynamic zero-sum games, instant games optimized for time, and so on. It is important to keep in mind differential games with partial information while participating in games in which the development of tactics is dependent on the passage of continuous time. In light of the numerous studies that have been conducted in this area of research, we are interested in articles that investigate various aspects of differential games and also models of dynamics.

Possible themes include, but are not limited to, the following:

- Differential games of pursuit and evasion;
- Games for dynamic equations on time scales;
- Two-person zero sum differential games;
- Linear-quadratic differential games;
- Dynamic games;
- Differential games described by PDE;
- Applications of differential games to biology, computer science, economics, engineering, management science, operations research, and political science;
- Deterministic and stochastic differential games with partial observation;

- Links between incomplete information games in continuous time and repeated time games;
- Hamilton–Jacobi equations for incomplete information games;
- Continuous time games;
- Pursuit and evasion differential games with incomplete information;
- Differential games on graphs;
- Hamilton–Jacobi equations in optimal control and differential games;
- Stochastic differential games;
- Evolutionary games;
- Differential games described by infinite system of differential equations;
- Numerical methods for differential games.

After evaluating submissions, we decided to publish seven papers in total.

Ibragimov et al. [1] investigate a differential version of the evasion game known as many pursuers and an evader, which takes place in the Hilbert space l_2 and is controlled by multiple infinite systems of two-block differential equations. The player control functions are restricted geometrically as a result of this limitation. In the event that the state of a controlled system returns to its starting point if the chase is finished in a differential game at some point in time after the l_2 space at a finite point in time, then the game is said to have been completed. The intention of the evader is to thwart the efforts of the pursuers, whose objective is to move the state of at least one of the systems into the starting point of space l_2, whereas the pursuers' objective is to do the opposite. An adequate evasion condition may be derived from any of the players' starting situations, and then an evasion strategy can be developed for the player who is attempting to escape.

Luckraz et al. [2] consider the game of pursuit and evasion that, when played on graphs, is often referred to as the game of cops and robbers. This traditional form of the game has been thoroughly solved by Nowakowski and Winkler, who provided the precise class of graphs for which the pursuer may win the game (sometimes referred to as a cop-win). Nowakowski and Winkler's Theorem is invalidated in the situation in which the graph does not fulfill the dismantlability condition. In this study, they provide some conceptions of cop-win and robber-win graphs that are on the more simplistic side. In particular, they decide that there will be one police officer on the scene, and they investigate if there are any possible combinations of beginning circumstances that might result in the graph favoring either the policemen or the robbers. Their objective is to provide both structural characterizations and algorithms that are computable in polynomial time (P) and that can handle weak cop-win and weak robber-win graphs. To this end, they offer certain open problems connected to this starting condition problem.

Yang et al. [3] start with the question that has to be answered by social science of how human cooperation developed through time. In order to foster collective collaboration and find solutions to social conundrums, one must first have a comprehensive knowledge of the human capacity for constrained rationality. They develop an asymmetric micro-dynamic that is based on limited rationality from a micro-viewpoint here by merging behavioral economics and cognitive psychology with evolutionary game theory. This allows us to look at the problem from a micro-perspective. Individuals were placed on a square lattice in order to participate in a voluntary public goods game in the Monte Carlo simulations that were run using asynchronously updated Monte Carlo models. The findings demonstrated that behaviors associated with free riding may, in the majority of cases, be successfully curbed. It is evident that the amount of collaboration may be improved in a population that consists of cooperators who are readily pleased and defectors who are greedy. Furthermore, crucial criteria for the stability of the system are further examined at the microscopic level, and altruistic conduct may be explained by the fact that a person who has lower expectations for or an underestimating of the potential outcomes of a single game is more likely to collaborate. Their position is that, in comparison to more conventional methods, the integration of ideas from other fields ought to be given greater weight and consideration.

Ibragimov et al. [4] carry out on an icosahedron in $\mathbb{R}^3$ an investigation of a differential game with m, $3 \leq m \leq 6$, pursuers, and one evader. When the maximum speeds of the pursuers are less than the speed of the evader, all of the players will travel down the 1-skeleton graph of the icosahedron. When the condition of the pursuer and the state of the evader coincide at any point during the pursuit, the pursuit is said to have been successful. In order to fulfill the need for the completion of the quest, they provide a suitable condition. Additionally, it is quite intriguing when there are obstructions around the edges. Particularly, a pursuer with a speed of zero might be thought of as such a barrier. Such a pursuer is immobile, but the chase is over when the evader's condition matches the pursuer's.

Ciano et al. [5] provide a generalization of the Brander and Taylor delayed continuous-time model for small islands introduced by Matsumoto et al. [6]. The dynamics may be simplified down to a set of ordinary differential equations if time lags are described using distributed delays rather than discrete delays. For the sake of making their analysis more straightforward, the gamma distribution function has been interpreted as having just weak and strong kernels. In each of the four distinct scenarios, the necessary criteria for the existence and maintainability of equilibria have been developed. In contrast to the delayed model, it has been discovered that the positive equilibrium becomes unstable for all high delay values. Alternatively, the stability of the equilibrium flips back, leading to the occurrence of repeated stability shifts.

Luckraz and Pansera [7] propose a no redundant information sets characteristic that, by using the idea of informative digraphs, may describe the precise class of extended games that can be temporally structured. This property can be characterized by the phrase "no duplicate information sets". Their findings may be used in the process of defining time-dependent solution ideas, such as the Open-Loop and the Closed-Loop Nash Equilibrium, in lengthy games including incomplete knowledge.

Ibragimov et al. [8] show a chase differential game for an infinite system of two-block differential equations in the Hilbert space l_2. The control functions for both the pursuer and the evader are bound by integral restrictions. When the state of the system at some finite point in time falls into the origin of the differential game, this is considered to be a successful conclusion to the differential game. While the pursuer's objective is to obtain the controlled system's state back to the beginning of space l_2, the evader's objective is to prevent this from happening. They come up with an equation to determine the ideal amount of time to spend pursuing, and they build the best possible strategy for the participants.

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